

# Discrimination of correlated and entangling quantum channels with selective process tomography

## Eugene Dumitrescu

**QMATH 2016**

Monday Oct 10 2016

Phys. Rev. A 92, 052329 (2015) — arXiv:1508.03053

Phys. Rev. A: Out this week (2016) — arXiv:1608.01416

Collaborators: Travis Humble



# Outline

1. Direct process tomography with stabilizer codes
2. Concatenation for error detection
3. Applications to channel discrimination
4. Conclusions and Future Directions

# Quantum Codes

# Quantum Codes

## Quantum Code:

Partitioning of Hilbert space

$$\mathcal{S} = \langle \hat{g}_1, \hat{g}_2, \dots, \hat{g}_{n-k} \rangle$$

$$\mathcal{C} = \{ |\psi\rangle \text{ s.t. } g_i |\psi\rangle = |\psi\rangle \ \forall i \}$$



# Quantum Codes

## Quantum Code:

Partitioning of Hilbert space

$$\mathcal{S} = \langle \hat{g}_1, \hat{g}_2, \dots, \hat{g}_{n-k} \rangle$$

$$\mathcal{C} = \{ |\psi\rangle \text{ s.t. } g_i |\psi\rangle = |\psi\rangle \ \forall i \}$$

## Quantum Error Correction:

Procedure to “combat” decoherence

$$\{\mathbb{E}_i\} \in \mathcal{P}_n = \{I, X, Y, Z\}^{\otimes n}$$

$$\text{s.t. } \langle \psi_i | \mathbb{E}_a \mathbb{E}_b | \psi_j \rangle = \delta_{ij} \delta_{ab}$$

# Quantum Codes

## Quantum Code:

Partitioning of Hilbert space

$$\mathcal{S} = \langle \hat{g}_1, \hat{g}_2, \dots, \hat{g}_{n-k} \rangle$$

$$\mathcal{C} = \{ |\psi\rangle \text{ s.t. } g_i |\psi\rangle = |\psi\rangle \ \forall i \}$$

## Quantum Error Correction:

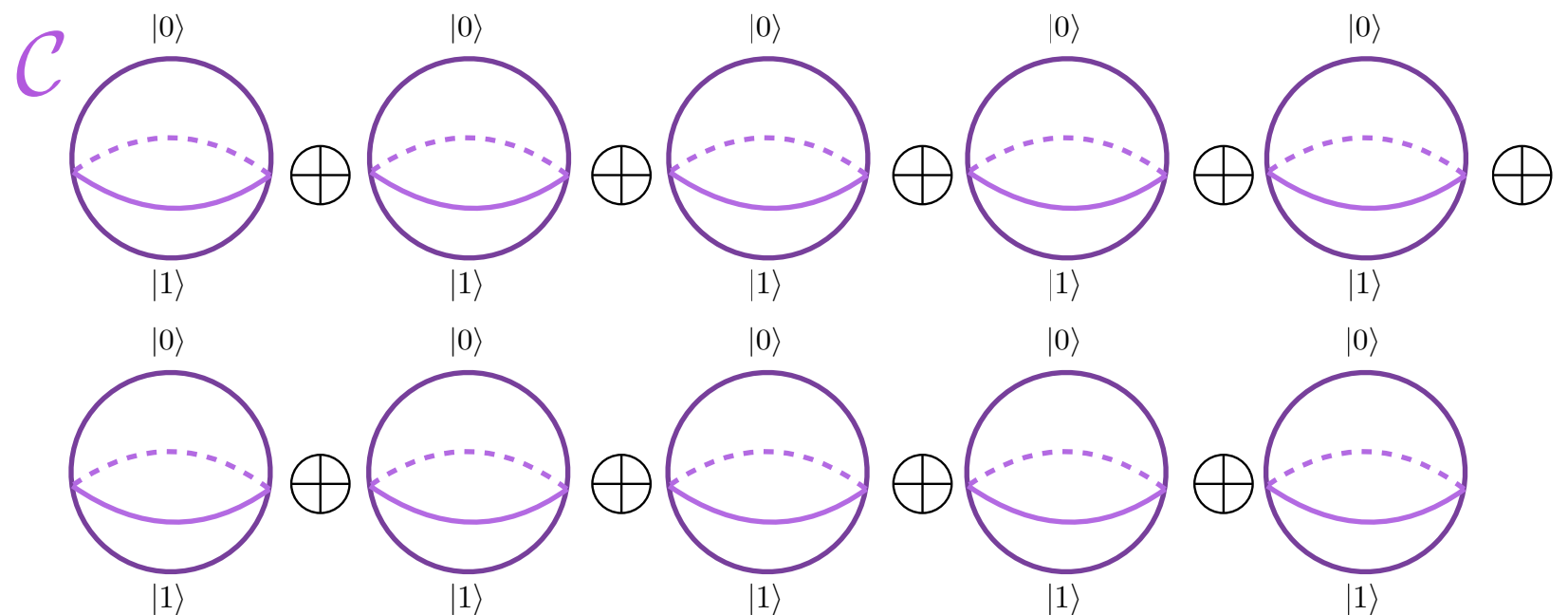
Procedure to “combat” decoherence

$$\{\mathbb{E}_i\} \in \mathcal{P}_n = \{I, X, Y, Z\}^{\otimes n}$$

$$\text{s.t. } \langle \psi_i | \mathbb{E}_a \mathbb{E}_b | \psi_j \rangle = \delta_{ij} \delta_{ab}$$

$$\begin{aligned} |0\rangle = & |0000000000\rangle + |0011110000\rangle \\ & + |0101010000\rangle + |0110100000\rangle \\ & + |1000111111\rangle + |1011001111\rangle \\ & + |1101101111\rangle + |1110011111\rangle \end{aligned}$$

$$\begin{aligned} |1\rangle = & |1111111111\rangle + |1100001111\rangle \\ & + |1010101111\rangle + |1001011111\rangle \\ & + |0111000000\rangle + |0100110000\rangle \\ & + |0010010000\rangle + |0001100000\rangle \end{aligned}$$



# Quantum Codes

## Quantum Code:

Partitioning of Hilbert space

$$\mathcal{S} = \langle \hat{g}_1, \hat{g}_2, \dots, \hat{g}_{n-k} \rangle$$

$$\mathcal{C} = \{ |\psi\rangle \text{ s.t. } g_i |\psi\rangle = |\psi\rangle \ \forall i \}$$

## Quantum Error Correction:

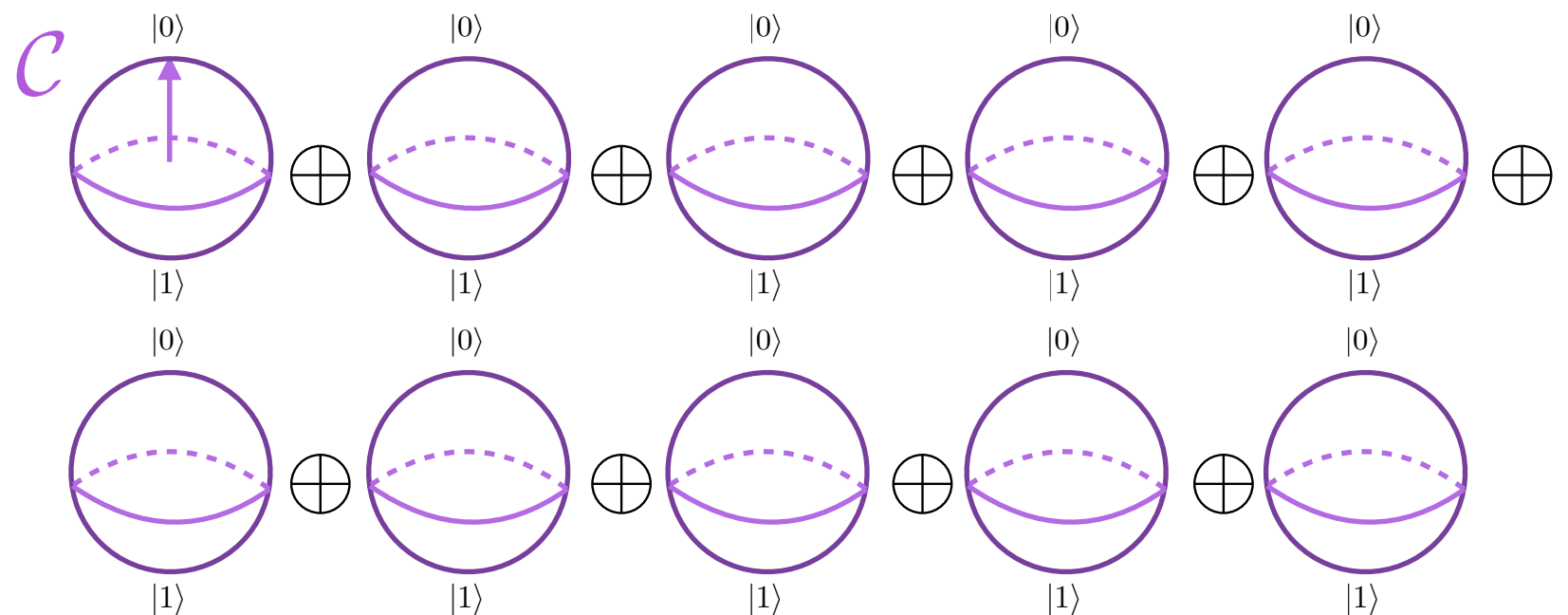
Procedure to “combat” decoherence

$$\{\mathbb{E}_i\} \in \mathcal{P}_n = \{I, X, Y, Z\}^{\otimes n}$$

$$\text{s.t. } \langle \psi_i | \mathbb{E}_a \mathbb{E}_b | \psi_j \rangle = \delta_{ij} \delta_{ab}$$

$$\begin{aligned} |0\rangle = & |0000000000\rangle + |0011110000\rangle \\ & + |0101010000\rangle + |0110100000\rangle \\ & + |1000111111\rangle + |1011001111\rangle \\ & + |1101101111\rangle + |1110011111\rangle \end{aligned}$$

$$\begin{aligned} |1\rangle = & |1111111111\rangle + |1100001111\rangle \\ & + |1010101111\rangle + |1001011111\rangle \\ & + |0111000000\rangle + |0100110000\rangle \\ & + |0010010000\rangle + |0001100000\rangle \end{aligned}$$



# Quantum Codes

## Quantum Code:

Partitioning of Hilbert space

$$\mathcal{S} = \langle \hat{g}_1, \hat{g}_2, \dots, \hat{g}_{n-k} \rangle$$

$$\mathcal{C} = \{ |\psi\rangle \text{ s.t. } g_i |\psi\rangle = |\psi\rangle \ \forall i \}$$

## Quantum Error Correction:

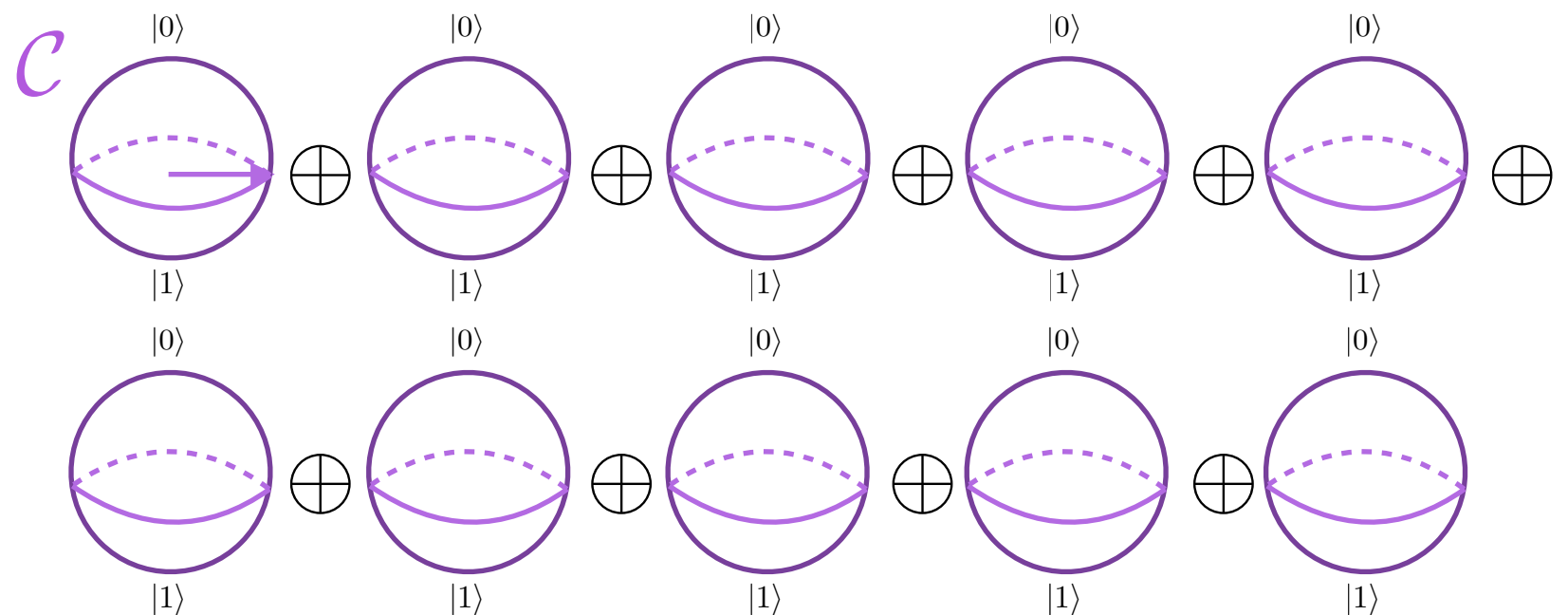
Procedure to “combat” decoherence

$$\{\mathbb{E}_i\} \in \mathcal{P}_n = \{I, X, Y, Z\}^{\otimes n}$$

$$\text{s.t. } \langle \psi_i | \mathbb{E}_a \mathbb{E}_b | \psi_j \rangle = \delta_{ij} \delta_{ab}$$

$$\begin{aligned} |0\rangle = & |0000000000\rangle + |0011110000\rangle \\ & + |0101010000\rangle + |0110100000\rangle \\ & + |1000111111\rangle + |1011001111\rangle \\ & + |1101101111\rangle + |1110011111\rangle \end{aligned}$$

$$\begin{aligned} |1\rangle = & |1111111111\rangle + |1100001111\rangle \\ & + |1010101111\rangle + |1001011111\rangle \\ & + |0111000000\rangle + |0100110000\rangle \\ & + |0010010000\rangle + |0001100000\rangle \end{aligned}$$



# Quantum Codes

## Quantum Code:

Partitioning of Hilbert space

$$\mathcal{S} = \langle \hat{g}_1, \hat{g}_2, \dots, \hat{g}_{n-k} \rangle$$

$$\mathcal{C} = \{ |\psi\rangle \text{ s.t. } g_i |\psi\rangle = |\psi\rangle \ \forall i \}$$

## Quantum Error Correction:

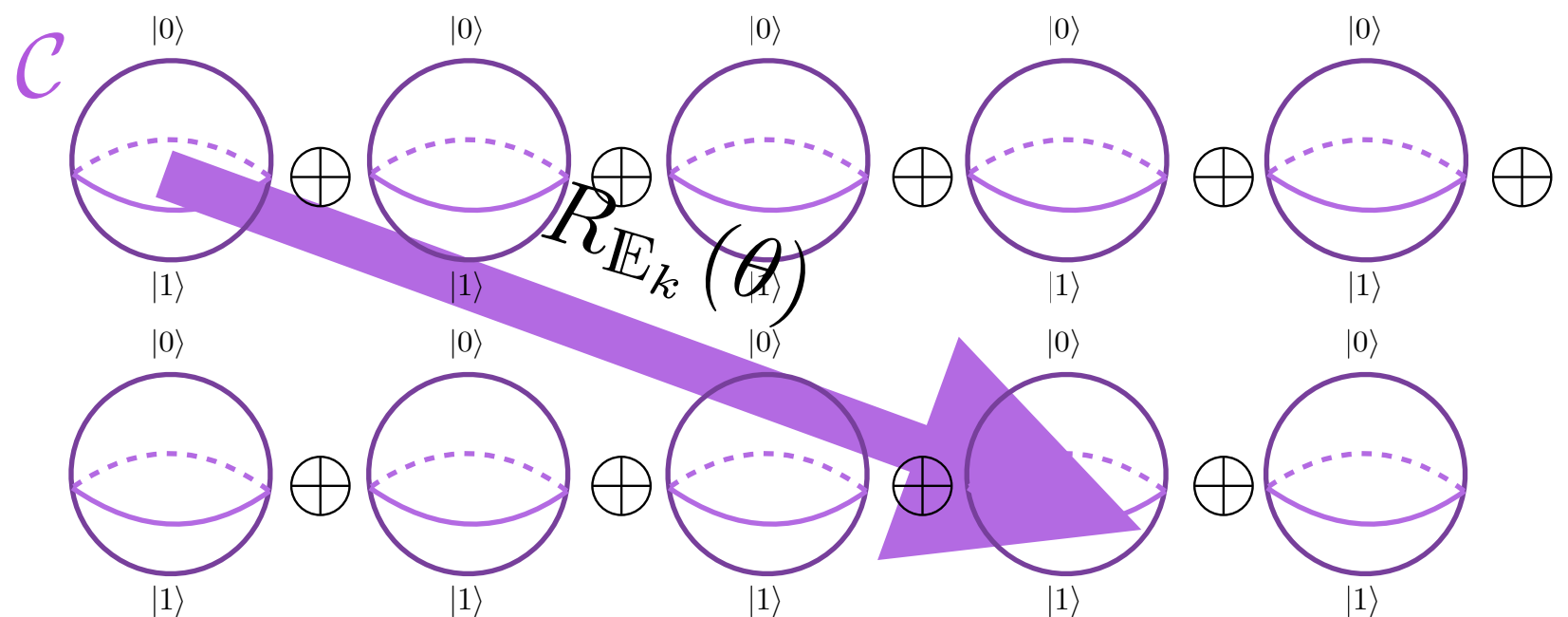
Procedure to “combat” decoherence

$$\{\mathbb{E}_i\} \in \mathcal{P}_n = \{I, X, Y, Z\}^{\otimes n}$$

$$\text{s.t. } \langle \psi_i | \mathbb{E}_a \mathbb{E}_b | \psi_j \rangle = \delta_{ij} \delta_{ab}$$

$$\begin{aligned} |0\rangle = & |0000000000\rangle + |0011110000\rangle \\ & + |0101010000\rangle + |0110100000\rangle \\ & + |1000111111\rangle + |1011001111\rangle \\ & + |1101101111\rangle + |1110011111\rangle \end{aligned}$$

$$\begin{aligned} |1\rangle = & |1111111111\rangle + |1100001111\rangle \\ & + |1010101111\rangle + |1001011111\rangle \\ & + |0111000000\rangle + |0100110000\rangle \\ & + |0010010000\rangle + |0001100000\rangle \end{aligned}$$



# Quantum Codes

## Quantum Code:

Partitioning of Hilbert space

$$\mathcal{S} = \langle \hat{g}_1, \hat{g}_2, \dots, \hat{g}_{n-k} \rangle$$

$$\mathcal{C} = \{ |\psi\rangle \text{ s.t. } g_i |\psi\rangle = |\psi\rangle \ \forall i \}$$

## Quantum Error Correction:

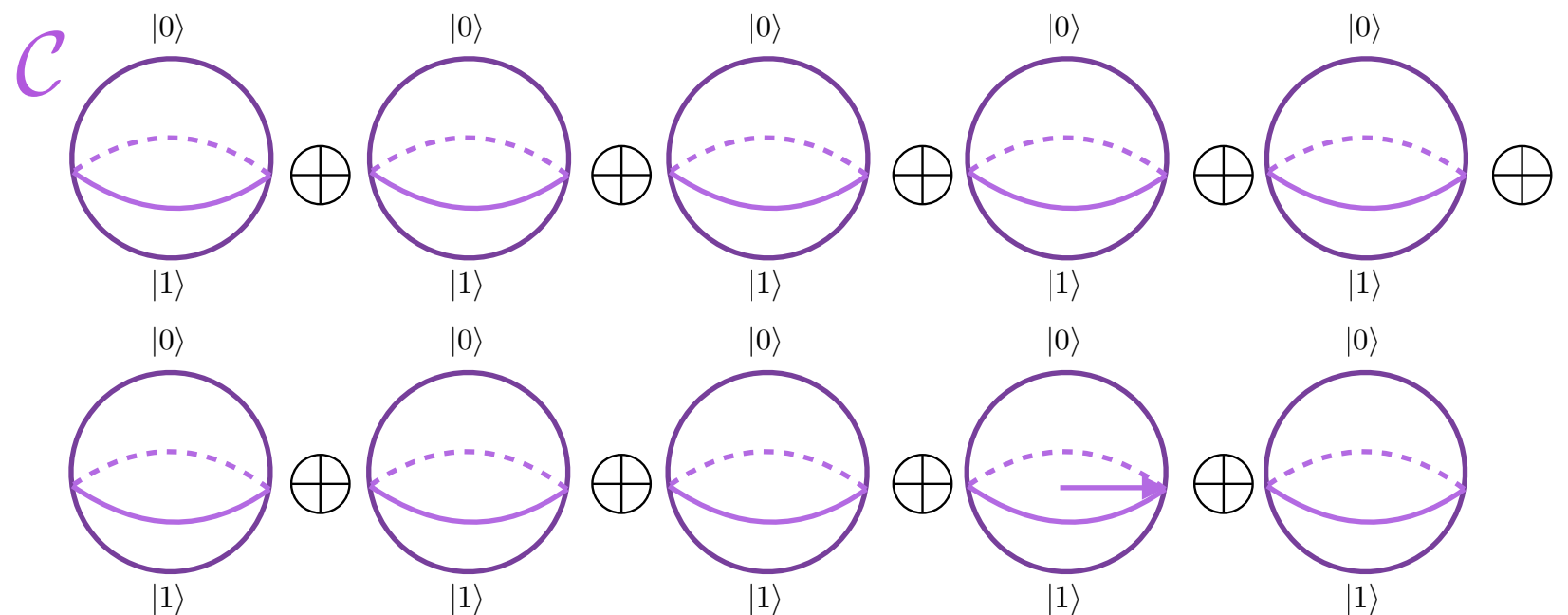
Procedure to “combat” decoherence

$$\{\mathbb{E}_i\} \in \mathcal{P}_n = \{I, X, Y, Z\}^{\otimes n}$$

$$\text{s.t. } \langle \psi_i | \mathbb{E}_a \mathbb{E}_b | \psi_j \rangle = \delta_{ij} \delta_{ab}$$

$$\begin{aligned} |0\rangle = & |0000000000\rangle + |0011110000\rangle \\ & + |0101010000\rangle + |0110100000\rangle \\ & + |1000111111\rangle + |1011001111\rangle \\ & + |1101101111\rangle + |1110011111\rangle \end{aligned}$$

$$\begin{aligned} |1\rangle = & |1111111111\rangle + |1100001111\rangle \\ & + |1010101111\rangle + |1001011111\rangle \\ & + |0111000000\rangle + |0100110000\rangle \\ & + |0010010000\rangle + |0001100000\rangle \end{aligned}$$



# Quantum Codes

## Quantum Code:

Partitioning of Hilbert space

$$\mathcal{S} = \langle \hat{g}_1, \hat{g}_2, \dots, \hat{g}_{n-k} \rangle$$

$$\mathcal{C} = \{ |\psi\rangle \text{ s.t. } g_i |\psi\rangle = |\psi\rangle \ \forall i \}$$

## Quantum Error Correction:

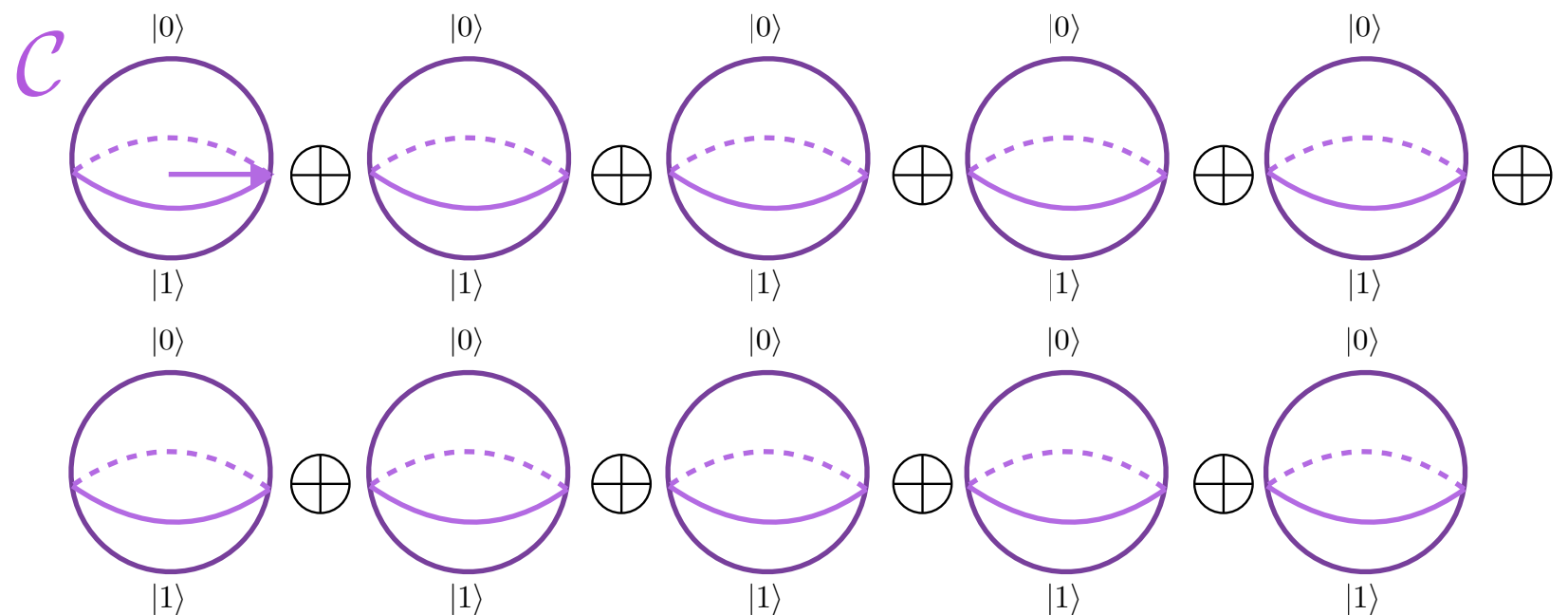
Procedure to “combat” decoherence

$$\{\mathbb{E}_i\} \in \mathcal{P}_n = \{I, X, Y, Z\}^{\otimes n}$$

$$\text{s.t. } \langle \psi_i | \mathbb{E}_a \mathbb{E}_b | \psi_j \rangle = \delta_{ij} \delta_{ab}$$

$$\begin{aligned} |0\rangle = & |0000000000\rangle + |0011110000\rangle \\ & + |0101010000\rangle + |0110100000\rangle \\ & + |1000111111\rangle + |1011001111\rangle \\ & + |1101101111\rangle + |1110011111\rangle \end{aligned}$$

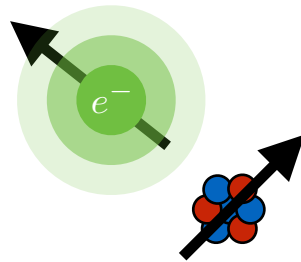
$$\begin{aligned} |1\rangle = & |1111111111\rangle + |1100001111\rangle \\ & + |1010101111\rangle + |1001011111\rangle \\ & + |0111000000\rangle + |0100110000\rangle \\ & + |0010010000\rangle + |0001100000\rangle \end{aligned}$$



# Quantum Channel



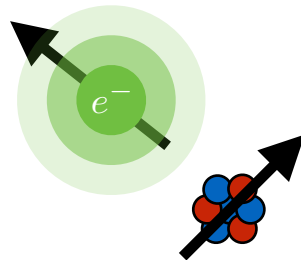
# Quantum Channel



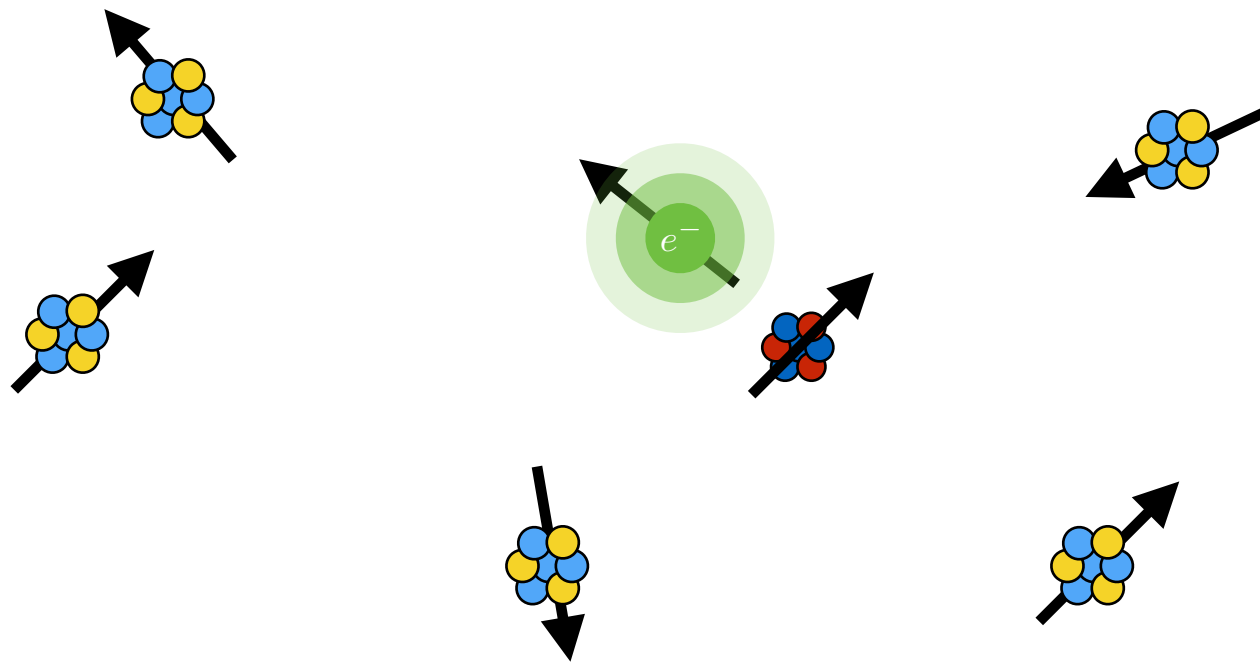
# Quantum Channel

$$i\hbar\partial_t\psi(t) = \hat{H}\psi(t)$$

$$\hat{H} = \hat{H}_{sys}$$

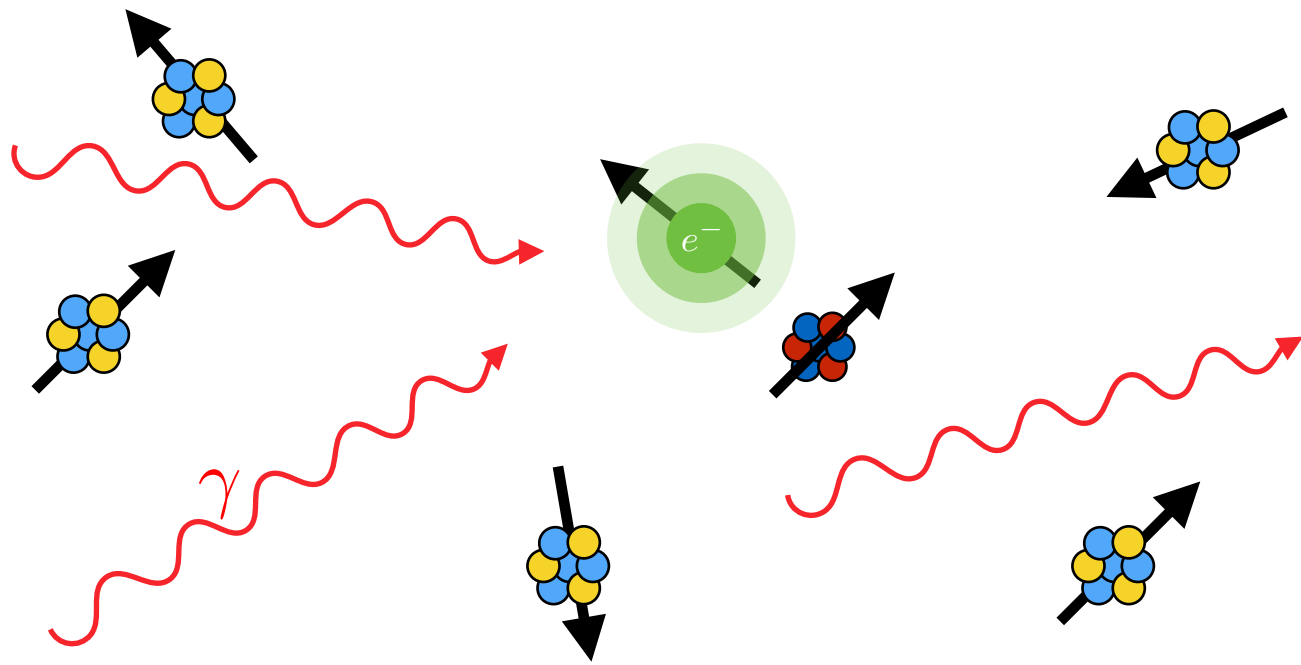


# Quantum Channel



$$i\hbar\partial_t\psi(t) = \hat{H}\psi(t)$$
$$\hat{H} = \hat{H}_{sys}$$
$$+ \hat{H}_{env} + \hat{H}_{sys-env}$$

# Quantum Channel

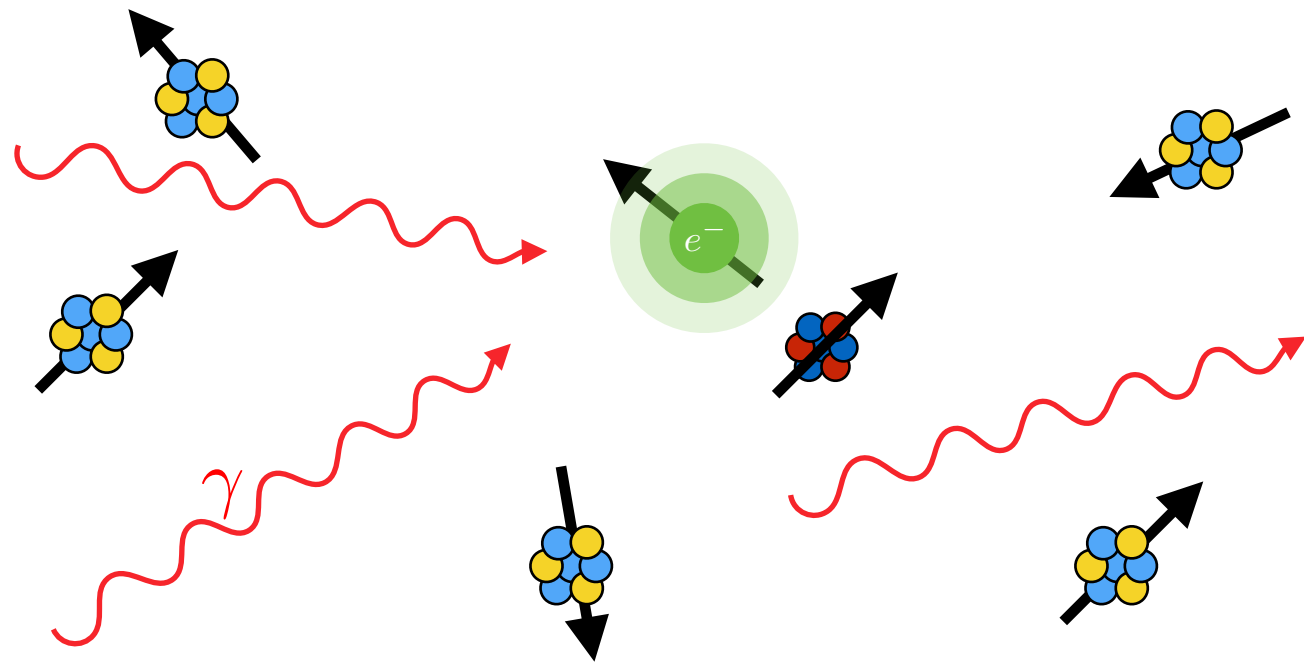


$$i\hbar\partial_t\psi(t) = \hat{H}\psi(t)$$

$$\hat{H} = \hat{H}_{sys}$$

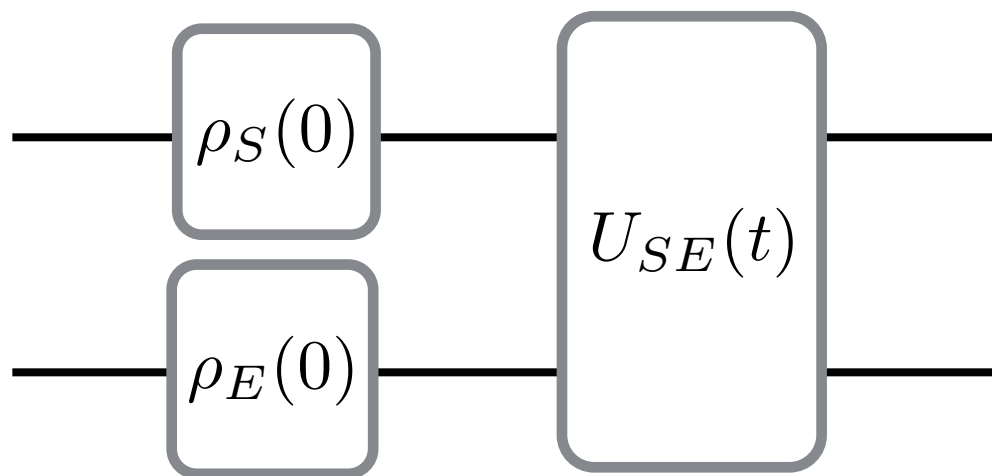
$$+ \hat{H}_{env} + \hat{H}_{sys-env}$$
$$+ \hat{H}_{cont} + \hat{H}_{sys-cont}$$
$$+ \hat{H}_{cont-env}$$

# Quantum Channel

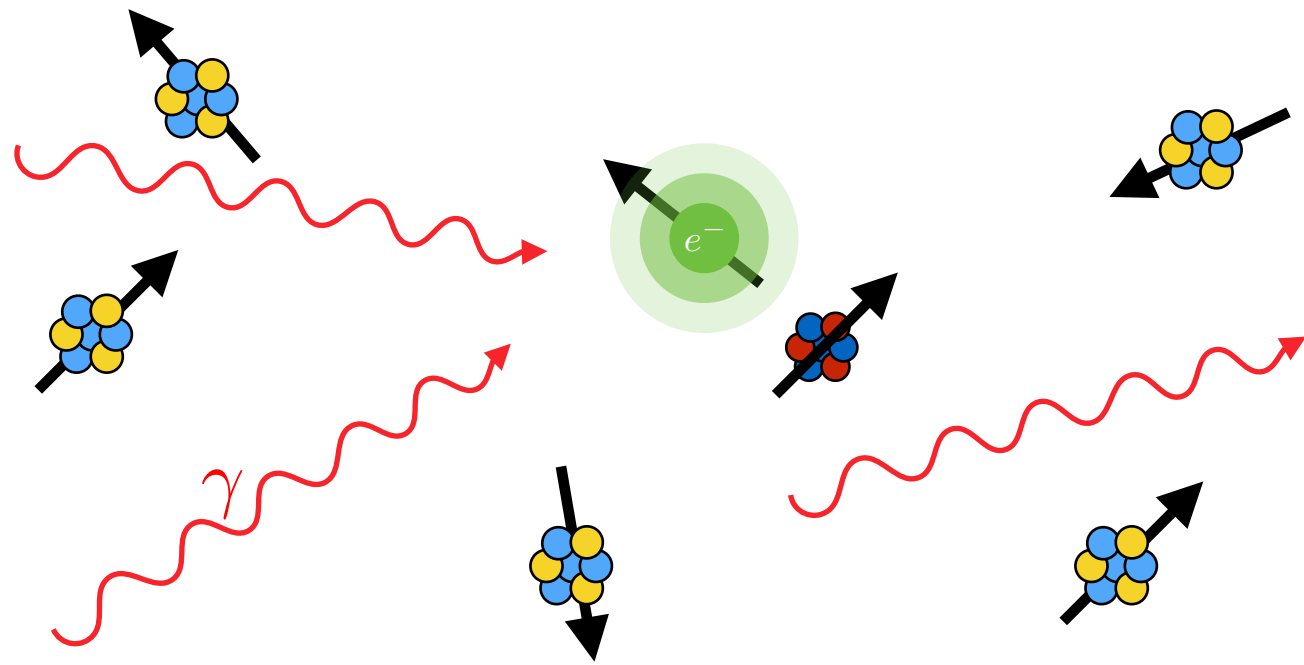


$$i\hbar\partial_t\psi(t) = \hat{H}\psi(t)$$

$$\begin{aligned}\hat{H} = & \hat{H}_{sys} \\ & + \hat{H}_{env} + \hat{H}_{sys-env} \\ & + \hat{H}_{cont} + \hat{H}_{sys-cont} \\ & + \hat{H}_{cont-env}\end{aligned}$$



# Quantum Channel



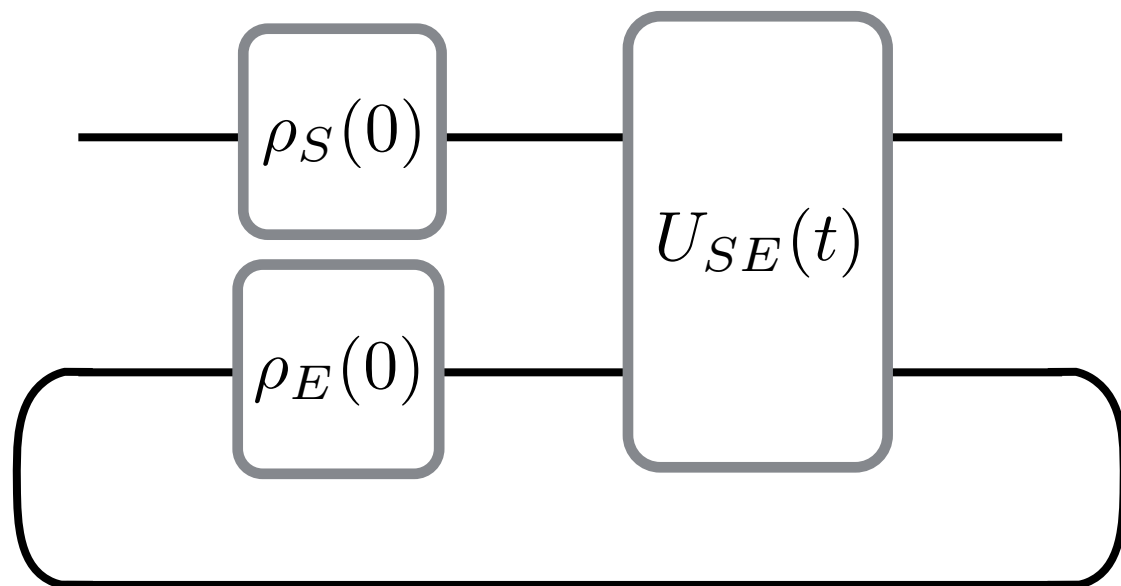
$$i\hbar\partial_t\psi(t) = \hat{H}\psi(t)$$

$$\hat{H} = \hat{H}_{sys}$$

$$+ \hat{H}_{env} + \hat{H}_{sys-env}$$

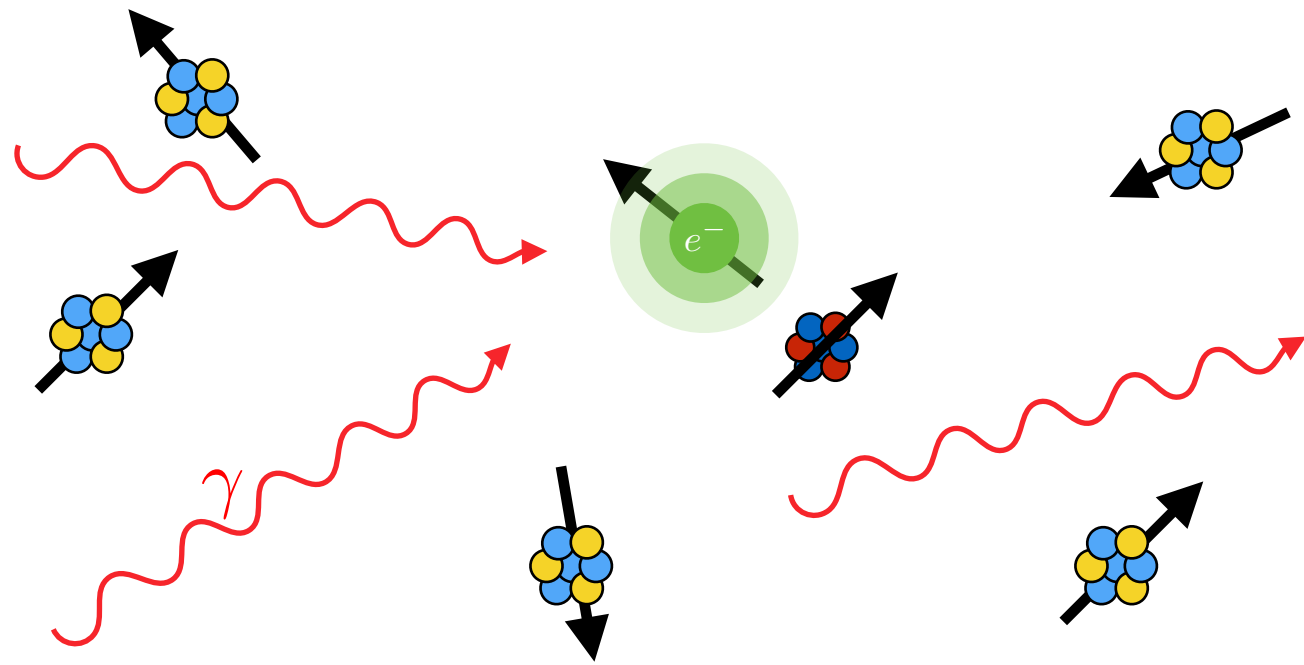
$$+ \hat{H}_{cont} + \hat{H}_{sys-cont}$$

$$+ \hat{H}_{cont-env}$$



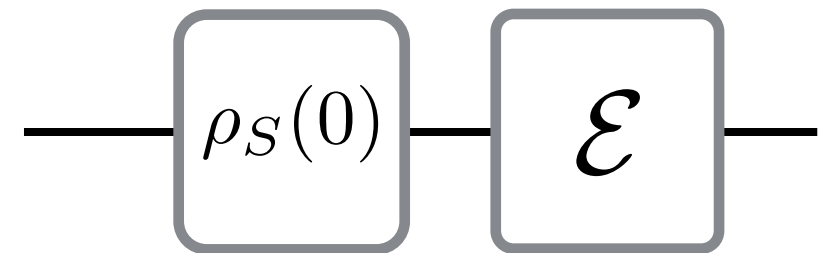
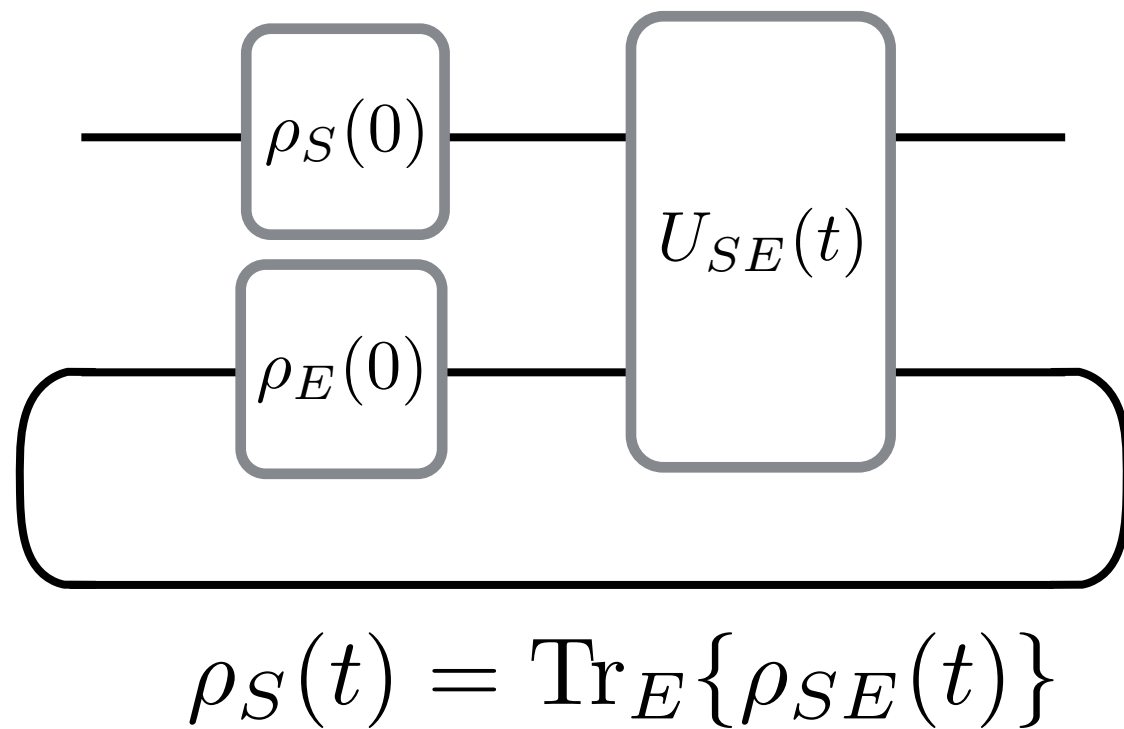
$$\rho_S(t) = \text{Tr}_E\{\rho_{SE}(t)\}$$

# Quantum Channel



$$i\hbar\partial_t\psi(t) = \hat{H}\psi(t)$$

$$\begin{aligned}\hat{H} = & \hat{H}_{sys} \\ & + \hat{H}_{env} + \hat{H}_{sys-env} \\ & + \hat{H}_{cont} + \hat{H}_{sys-cont} \\ & + \hat{H}_{cont-env}\end{aligned}$$

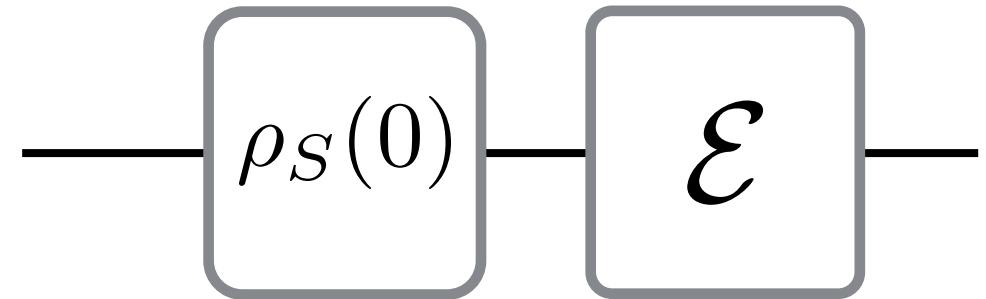


$$\mathcal{E} : \rho_i \mapsto \rho_f$$

$$\mathcal{E}(\rho_s) = \sum_i K_i \rho_s K_i^\dagger$$

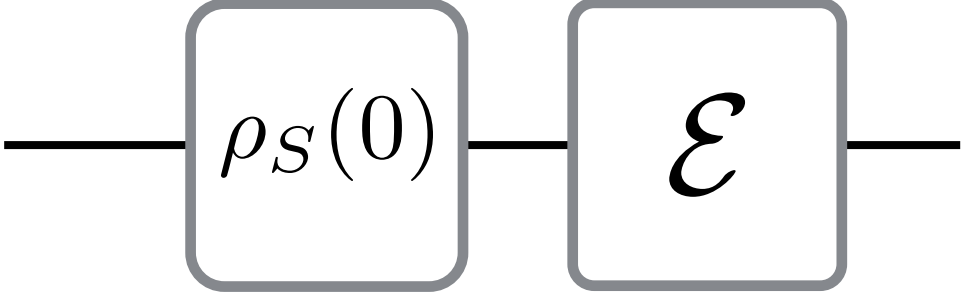
# Channel Structure

$$\mathcal{E}(\rho) = \sum_i K_i \rho K_i^\dagger$$





# Channel Structure

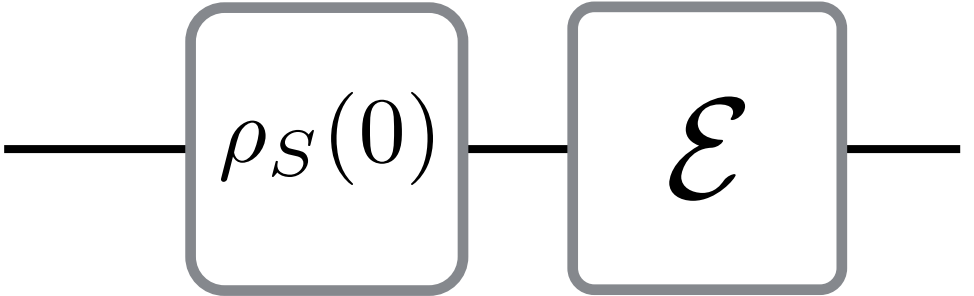
$$\mathcal{E}(\rho) = \sum_i K_i \rho K_i^\dagger$$


$\rho$  :  $d \times d$  positive semi-definite linear operator

$K_i$  :  $d \times d$  linear operators

$$K_i = \sum_{m=1}^{d^2} e_{im} F_m$$

# Channel Structure

$$\mathcal{E}(\rho) = \sum_i K_i \rho K_i^\dagger$$


$\rho$  :  $d \times d$  positive semi-definite linear operator

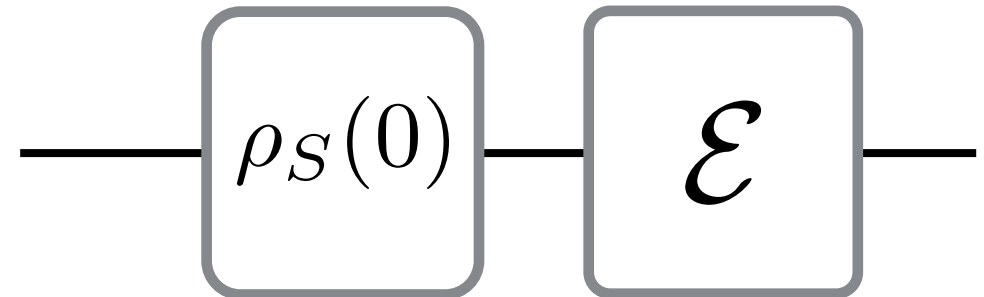
$K_i$  :  $d \times d$  linear operators

$$K_i = \sum_{m=1}^{d^2} e_{im} F_m$$

$$\mathcal{E}(\rho) = \sum_{m,n} \chi_{mn} F_m \rho F_n^\dagger$$

# Channel Structure

$$\mathcal{E}(\rho) = \sum_i K_i \rho K_i^\dagger$$



$\rho$  :  $d \times d$  positive semi-definite linear operator

$K_i$  :  $d \times d$  linear operators

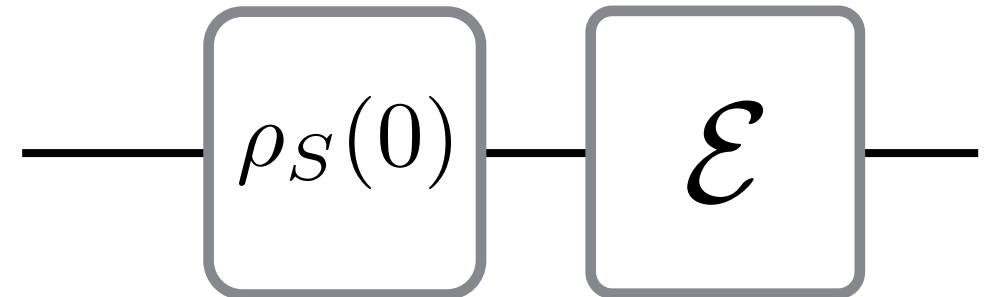
$$K_i = \sum_{m=1}^{d^2} e_{im} F_m$$

$$\mathcal{E}(\rho) = \sum_{m,n} \chi_{mn} F_m \rho F_n^\dagger$$

**Complete description**

# Channel Structure

$$\mathcal{E}(\rho) = \sum_i K_i \rho K_i^\dagger$$



$\rho$  :  $d \times d$  positive semi-definite linear operator

$K_i$  :  $d \times d$  linear operators

$$K_i = \sum_{m=1}^{d^2} e_{im} F_m$$

$$\mathcal{E}(\rho) = \sum_{m,n} \chi_{mn} F_m \rho F_n^\dagger$$

**Complete description**

## Informational Completeness

$$\chi_{mn} = \sum_i e_{im}^* e_{in}$$

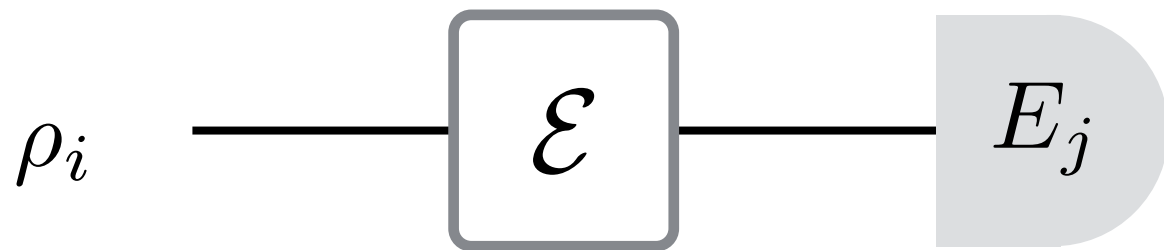
$$\sum_i K_i^\dagger K_i = 1 = \sum_{mn} \chi_{mn} F_m^\dagger F_n$$

$d^4$  elements -  $d^2$  constraints

$\mathcal{O}(d^4) = \mathcal{O}(16^n)$  measurements

# Channel Tomography

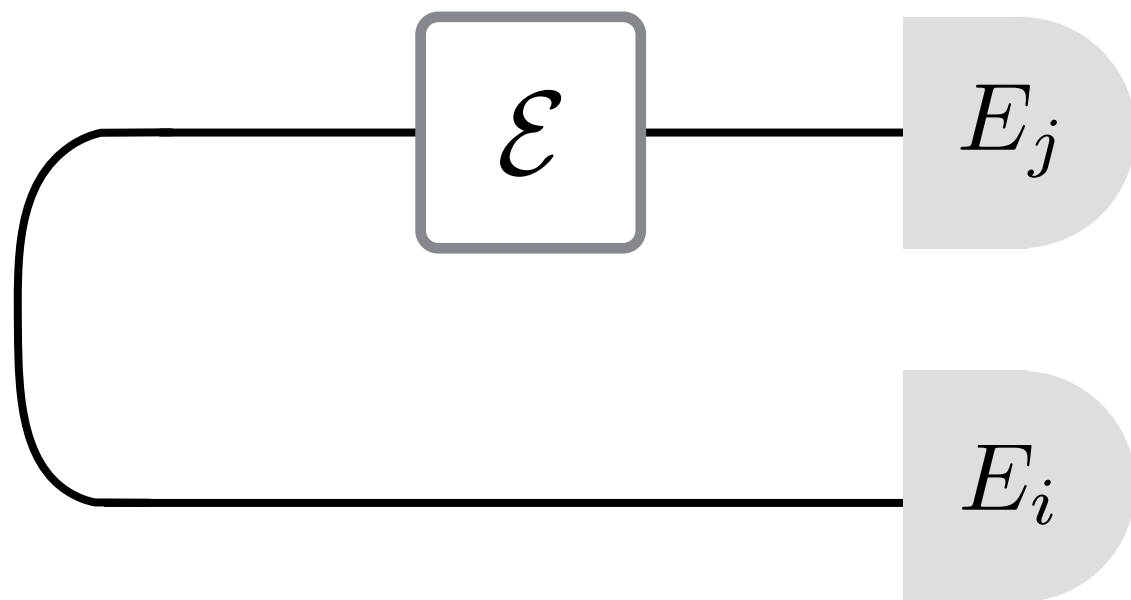
# Channel Tomography



$$\rho_i, E_j \in \{\Pi_0, \Pi_1, \Pi_+, \Pi_{+i}\}$$

**SQPT**

# Channel Tomography



$$\rho_i, E_j \in \{\Pi_0, \Pi_1, \Pi_+, \Pi_{+i}\}$$

**SQPT**

$$E_i, E_j \in \{\Pi_0, \Pi_1, \Pi_+, \Pi_{+i}\}$$

**AAPT**

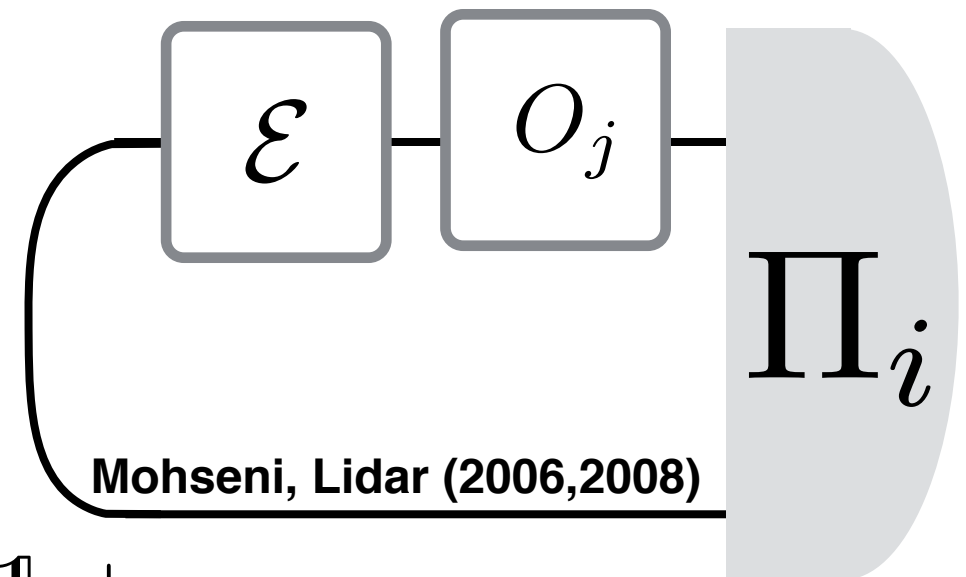
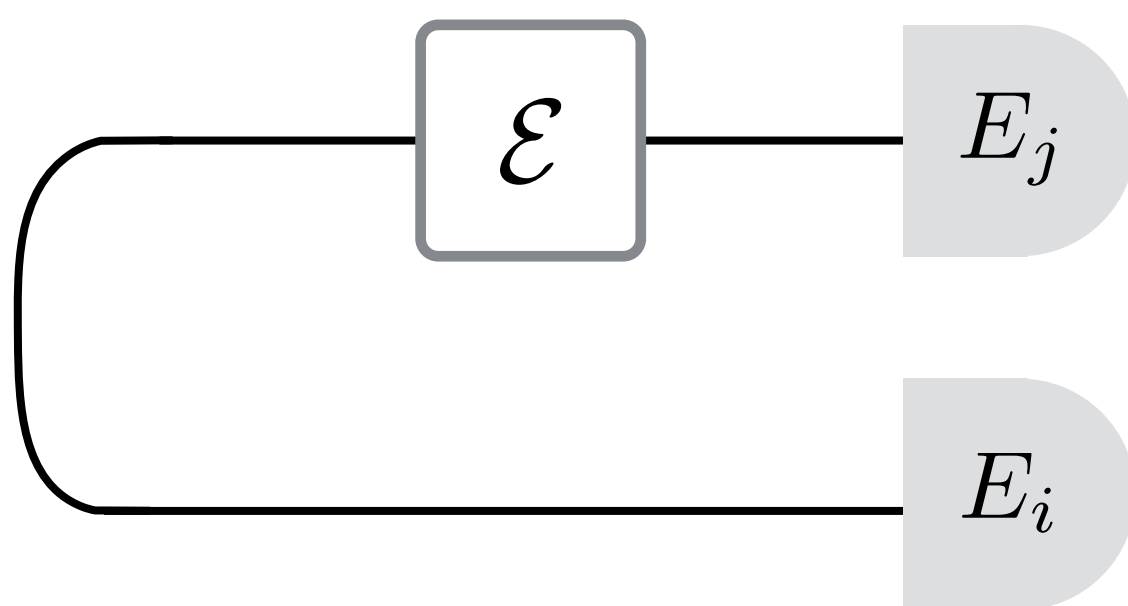
$$\Pi_i = \frac{\mathbb{1} + g_i}{2}; \quad \mathcal{S} = \langle g_1, g_2, \dots, g_{n-k} \rangle$$

$$\chi_{i,i} = \text{Tr}\{\Pi_i \mathcal{E}(\rho)\}$$

channel  $\sim$  state

$$\mathcal{E} \sim \mathcal{E} \otimes \mathbb{I}(\rho_{max})$$

# Channel Tomography



Mohseni, Lidar (2006,2008)

$$\rho_i, E_j \in \{\Pi_0, \Pi_1, \Pi_+, \Pi_{+i}\}$$

**SQPT**

$$E_i, E_j \in \{\Pi_0, \Pi_1, \Pi_+, \Pi_{+i}\}$$

**AAPT**

$$\Pi_i = \frac{\mathbb{1} + g_i}{2}; \quad \mathcal{S} = \langle g_1, g_2, \dots, g_{n-k} \rangle$$

$$\chi_{i,i} = \text{Tr}\{\Pi_i \mathcal{E}(\rho)\}$$

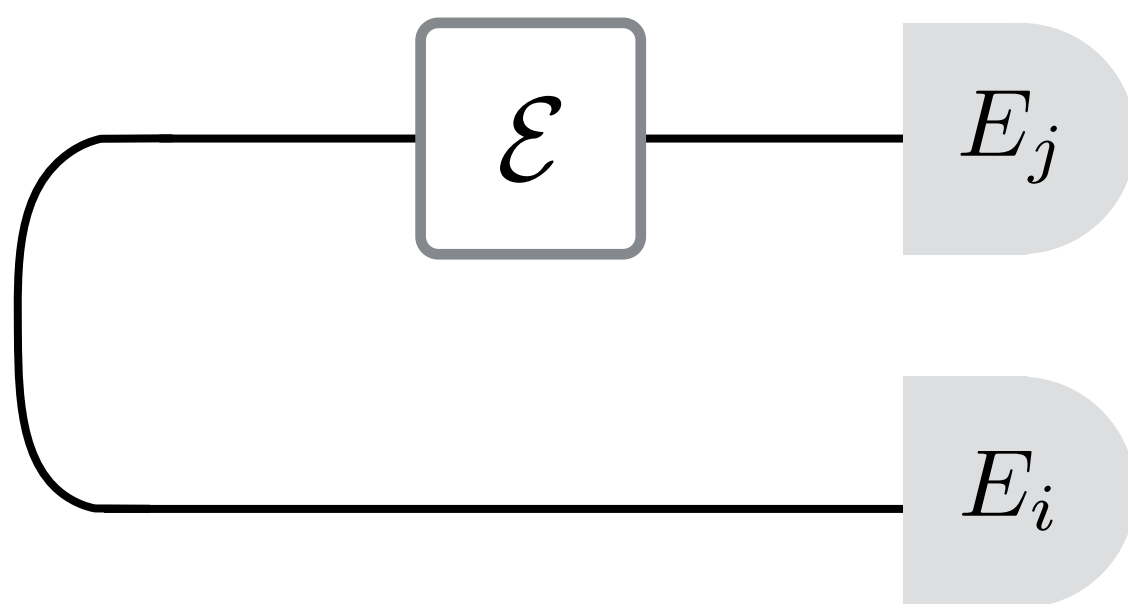
$$\chi_{i,j} = \text{Tr}\{\Pi_i O_j \mathcal{E}(\rho) O_j\}$$

channel  $\sim$  state

$$\mathcal{E} \sim \mathcal{E} \otimes \mathbb{I}(\rho_{max})$$



# Channel Tomography

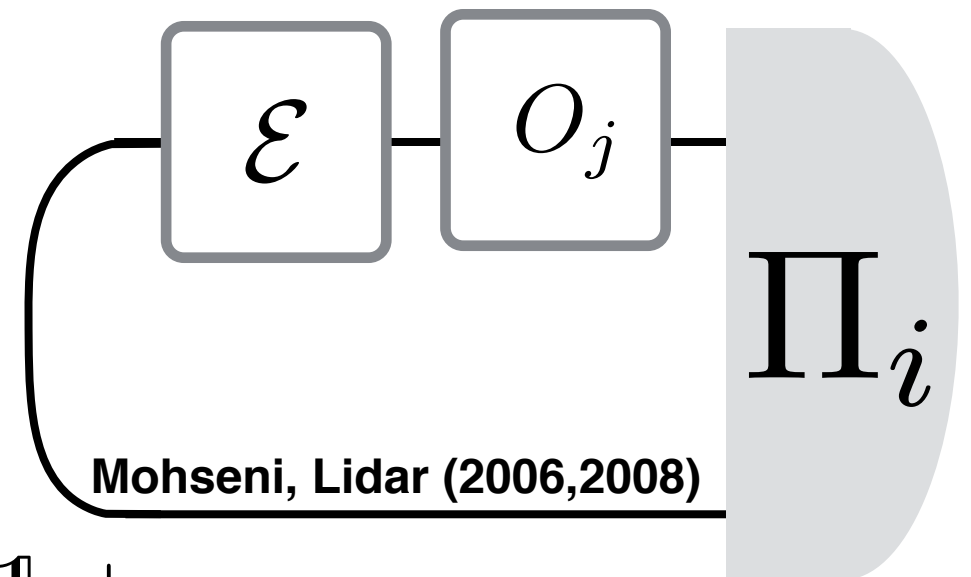
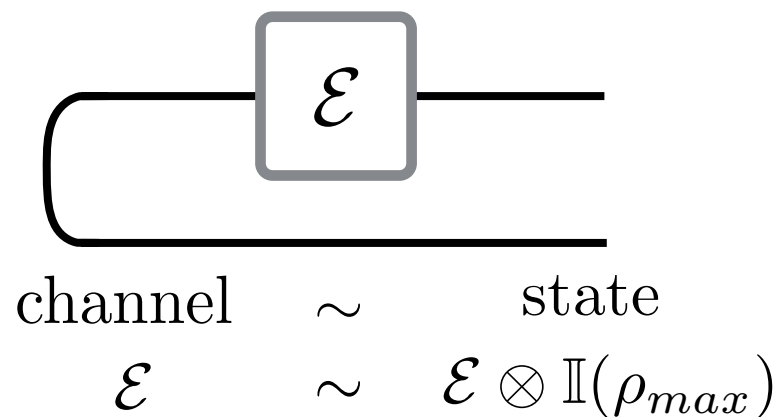


$$\rho_i, E_j \in \{\Pi_0, \Pi_1, \Pi_+, \Pi_{+i}\}$$

**SQPT**

$$E_i, E_j \in \{\Pi_0, \Pi_1, \Pi_+, \Pi_{+i}\}$$

**AAPT**

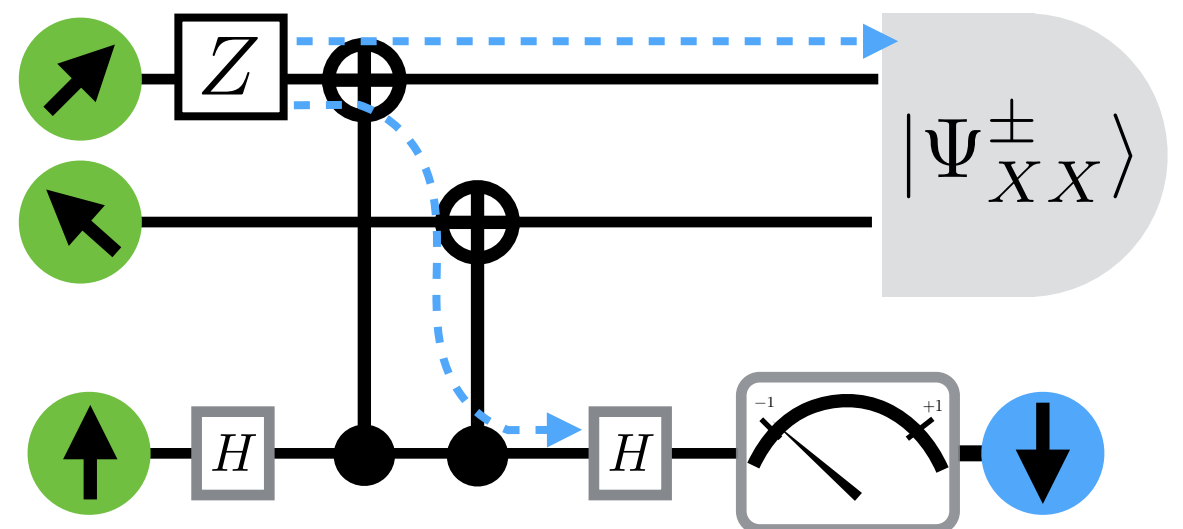


Mohseni, Lidar (2006,2008)

$$\Pi_i = \frac{\mathbb{1} + g_i}{2}; \quad \mathcal{S} = \langle g_1, g_2, \dots, g_{n-k} \rangle$$

$$\chi_{i,i} = \text{Tr}\{\Pi_i \mathcal{E}(\rho)\}$$

$$\chi_{i,j} = \text{Tr}\{\Pi_i O_j \mathcal{E}(\rho) O_j\}$$

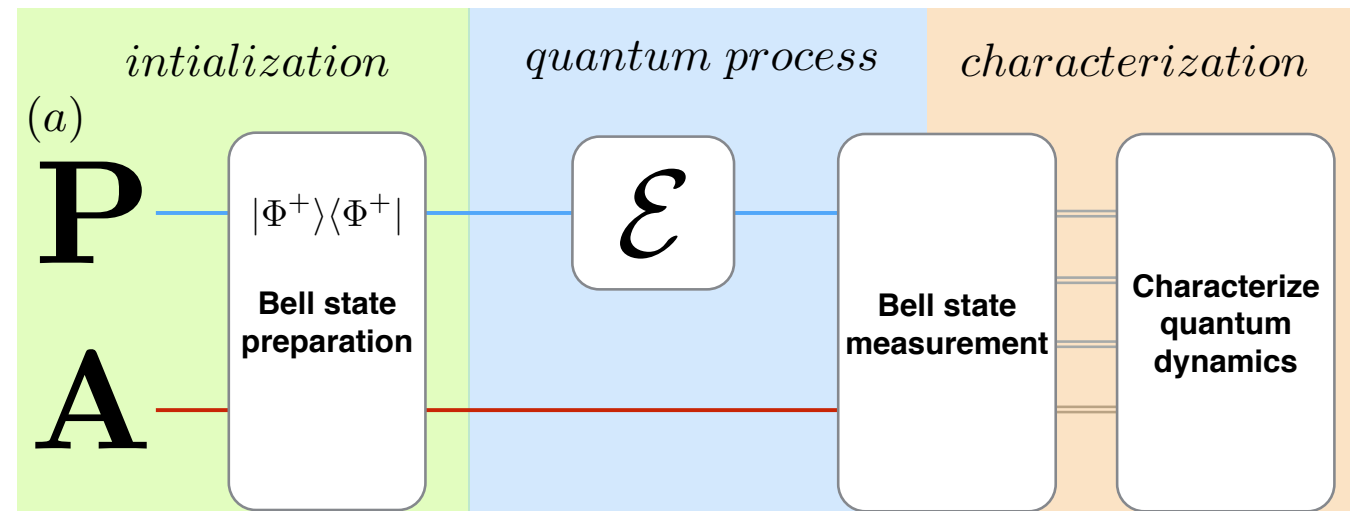


# Entanglement Assisted Codes

# Entanglement Assisted Codes

Codestate:  $|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$

Generators:  $\mathcal{S} = \langle XX, ZZ \rangle$

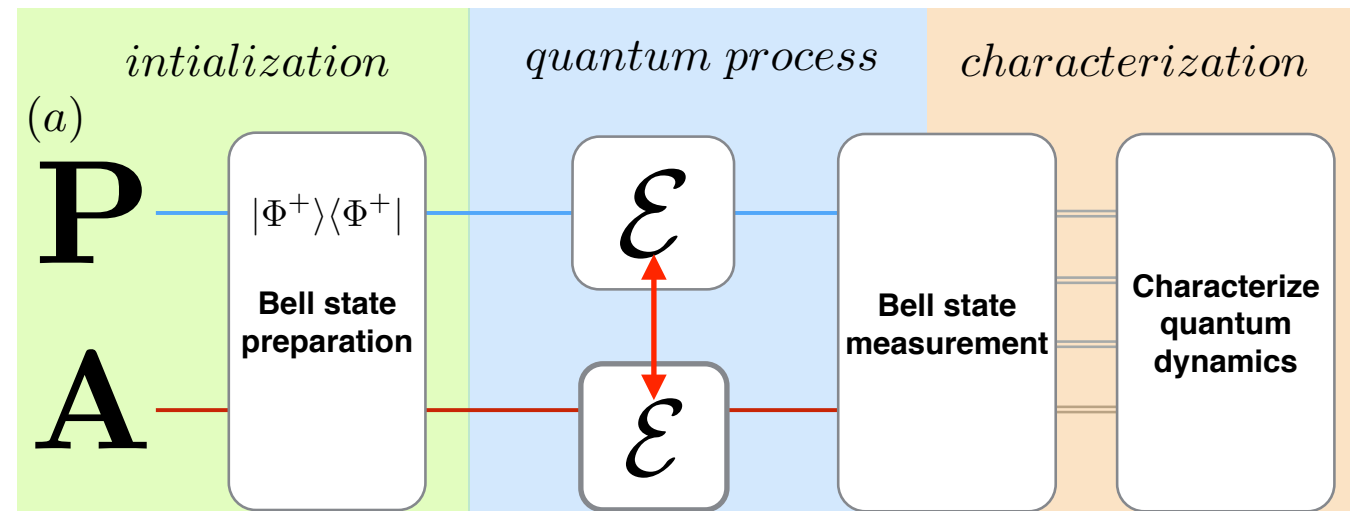


# Entanglement Assisted Codes

Codestate:  $|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$

Generators:  $\mathcal{S} = \langle XX, ZZ \rangle$

$Z_1, Z_2 : |\Phi^+\rangle \mapsto |\Phi^-\rangle$

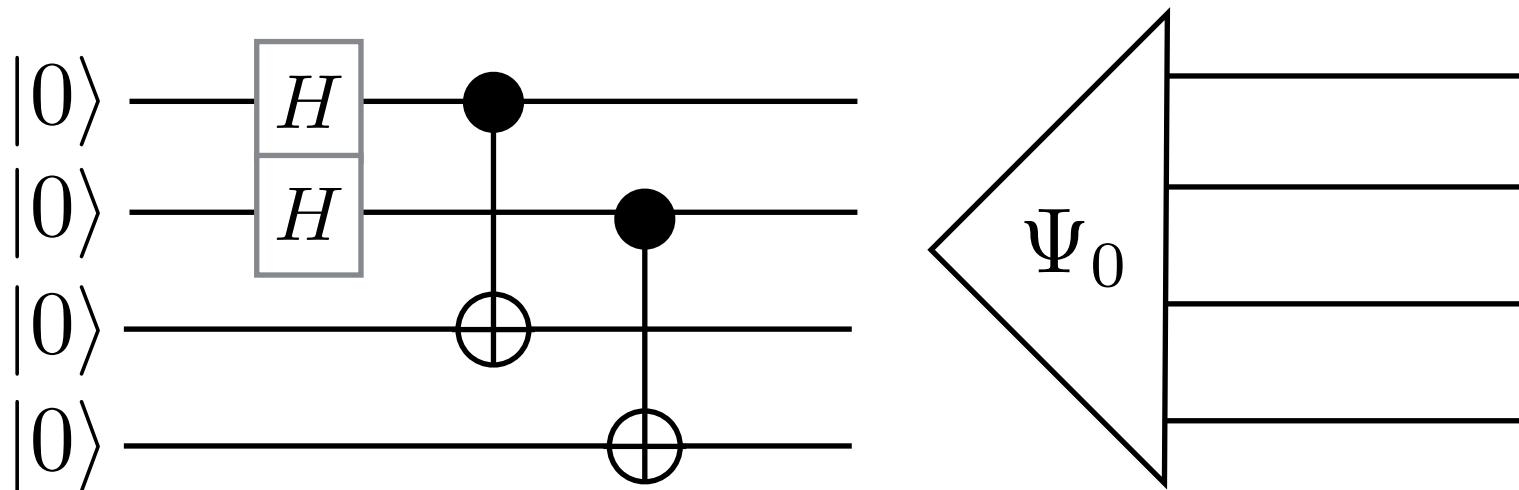
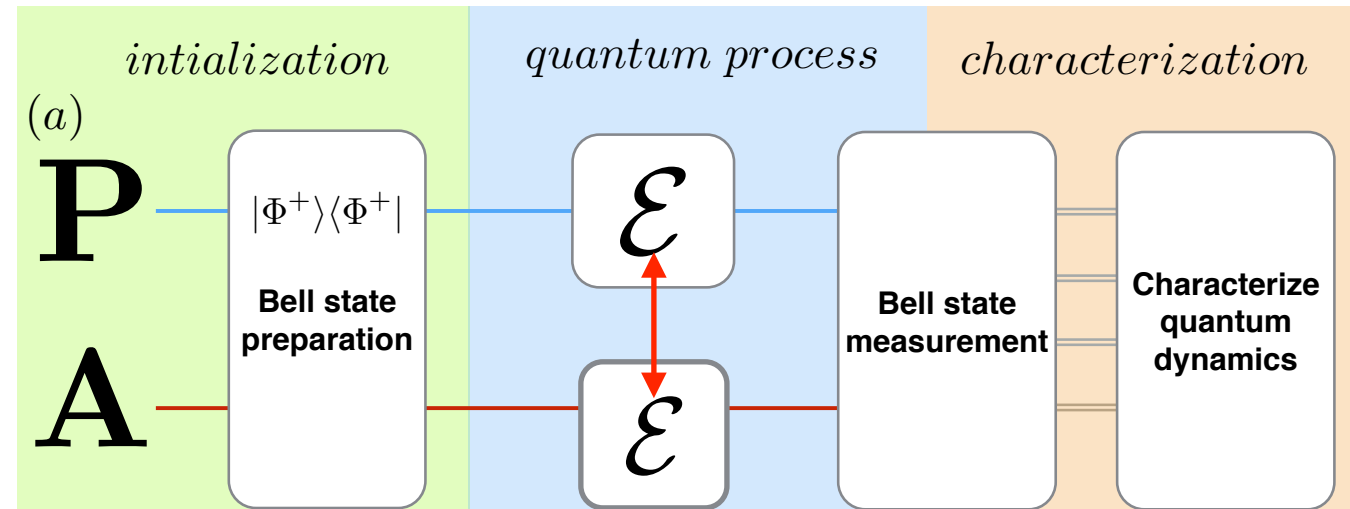


# Entanglement Assisted Codes

Codestate:  $|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$

Generators:  $\mathcal{S} = \langle XX, ZZ \rangle$

$Z_1, Z_2 : |\Phi^+\rangle \mapsto |\Phi^-\rangle$

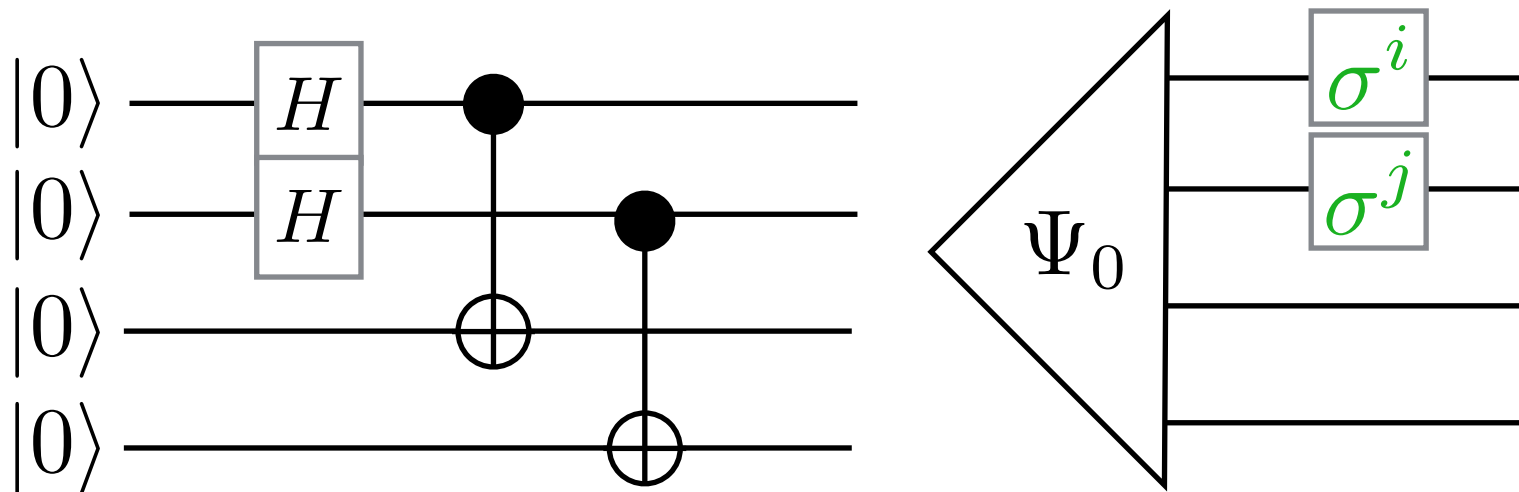
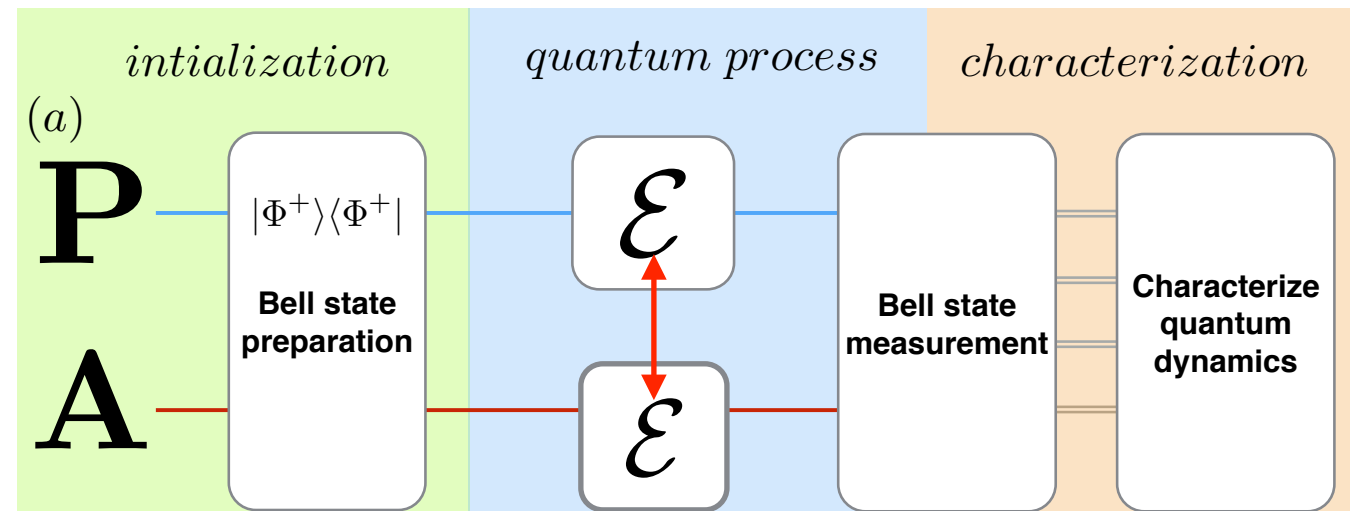


# Entanglement Assisted Codes

Codestate:  $|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$

Generators:  $\mathcal{S} = \langle XX, ZZ \rangle$

$Z_1, Z_2 : |\Phi^+\rangle \mapsto |\Phi^-\rangle$

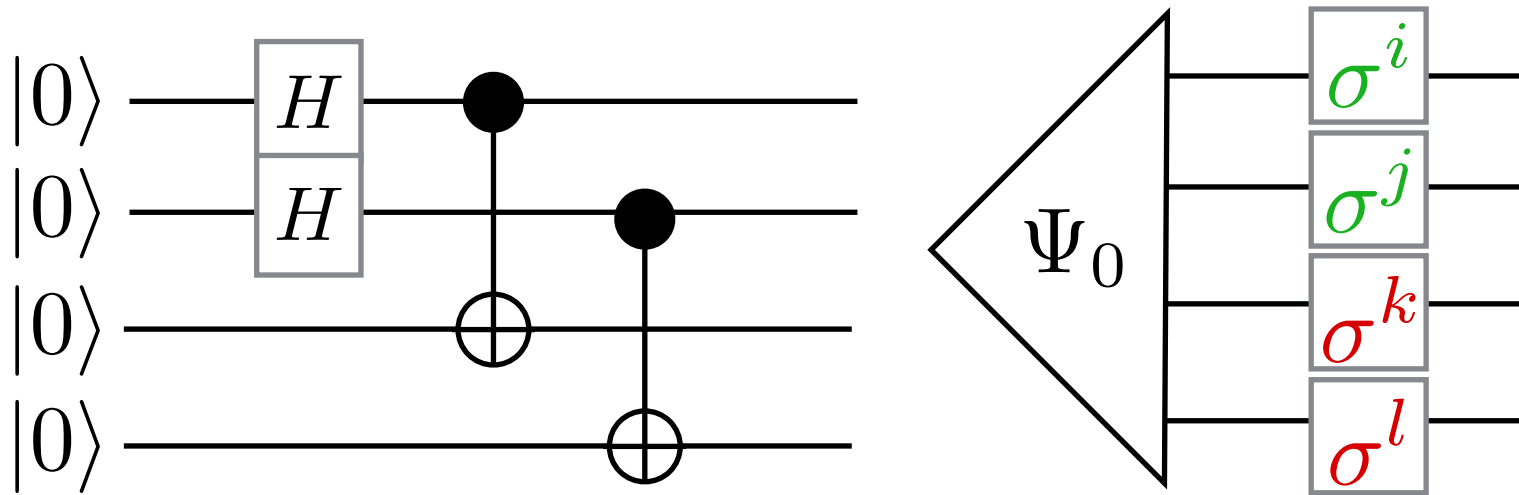
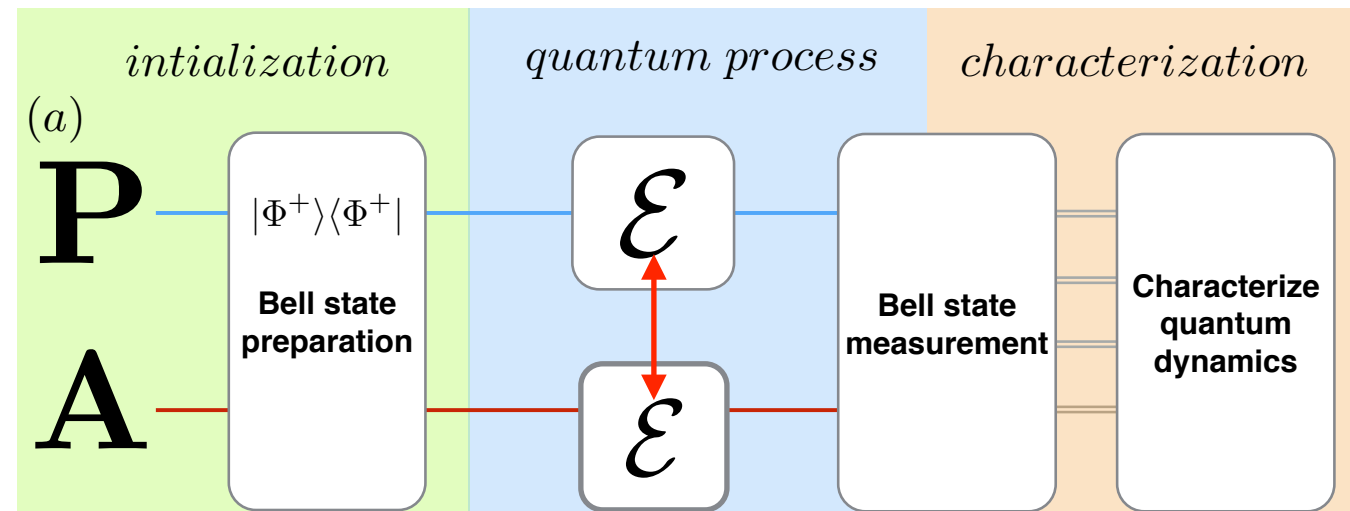


# Entanglement Assisted Codes

Codestate:  $|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$

Generators:  $\mathcal{S} = \langle XX, ZZ \rangle$

$Z_1, Z_2 : |\Phi^+\rangle \mapsto |\Phi^-\rangle$

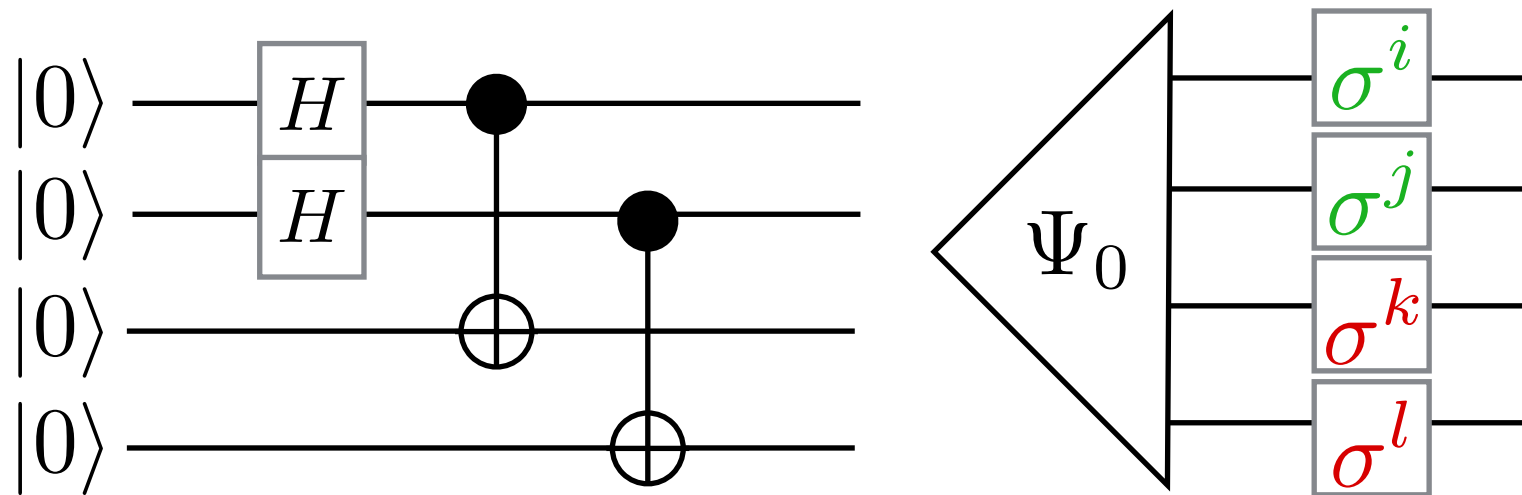
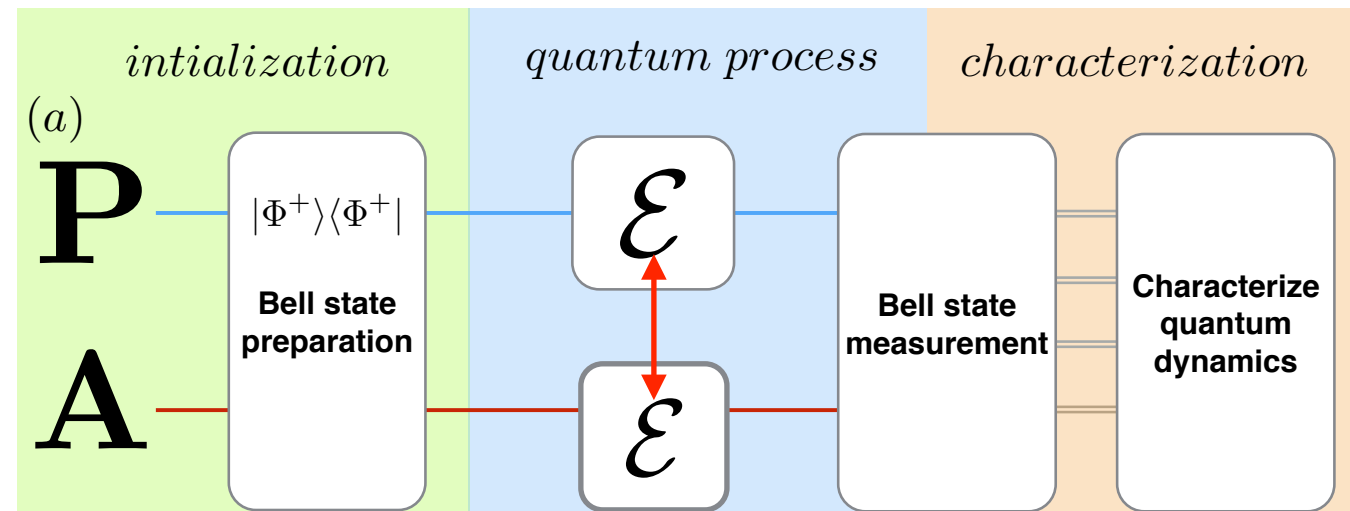


# Entanglement Assisted Codes

Codestate:  $|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$

Generators:  $\mathcal{S} = \langle XX, ZZ \rangle$

$Z_1, Z_2 : |\Phi^+\rangle \mapsto |\Phi^-\rangle$



Code invariant under

$$\mathbf{A} \Leftrightarrow \mathbf{P}$$

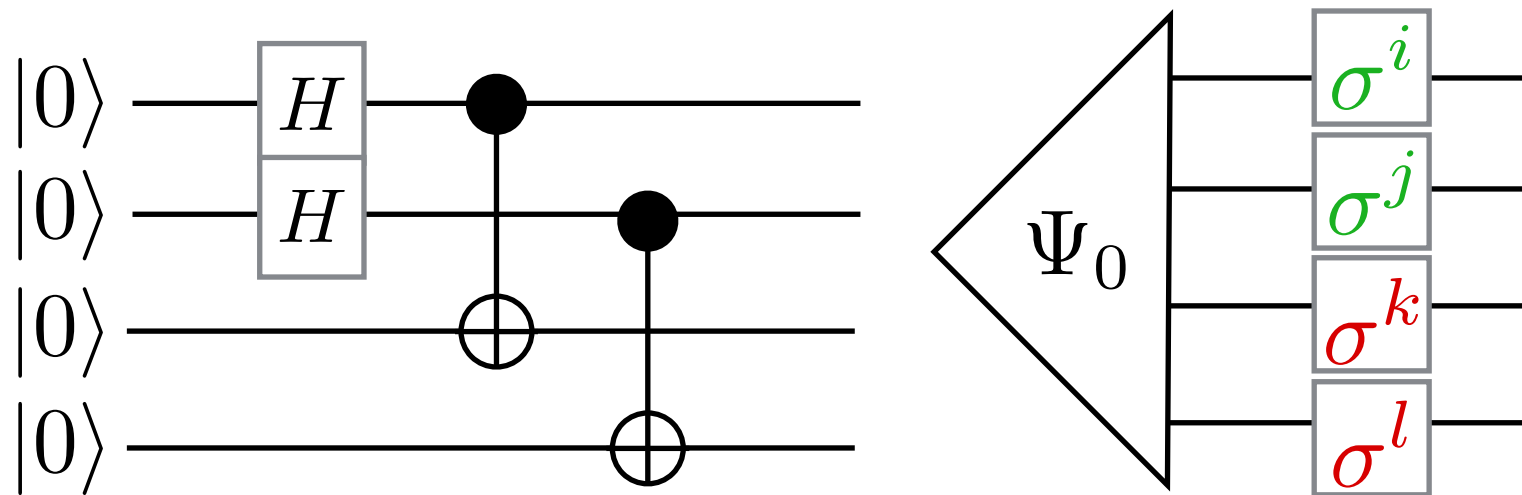
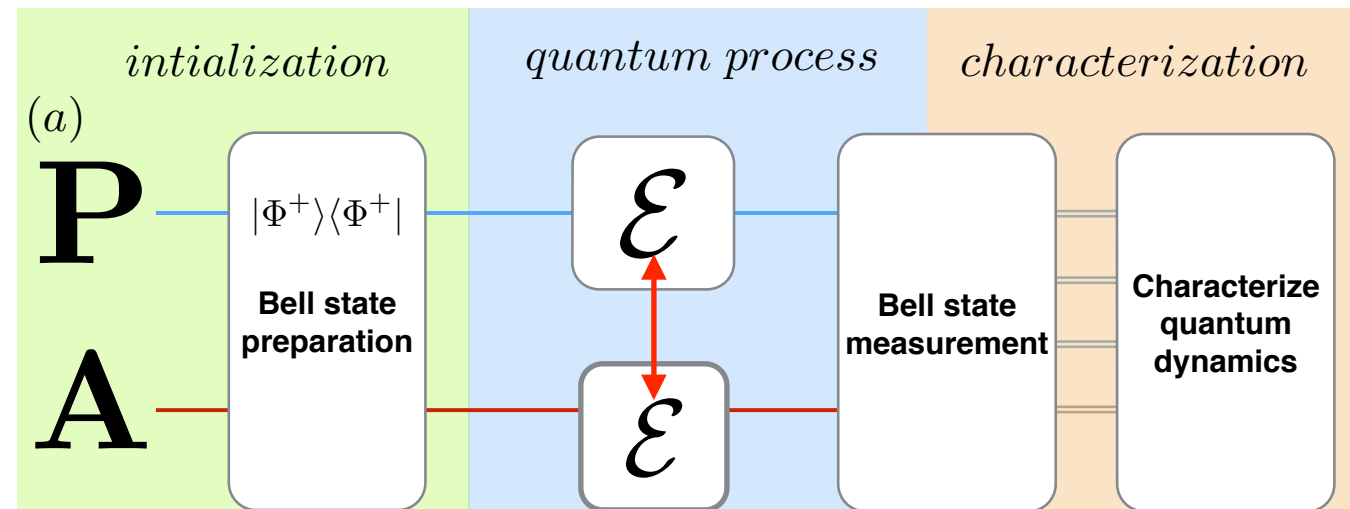


# Entanglement Assisted Codes

Codestate:  $|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$

Generators:  $\mathcal{S} = \langle XX, ZZ \rangle$

$Z_1, Z_2 : |\Phi^+\rangle \mapsto |\Phi^-\rangle$



Code invariant under

$$\mathbf{A} \Leftrightarrow \mathbf{P}$$

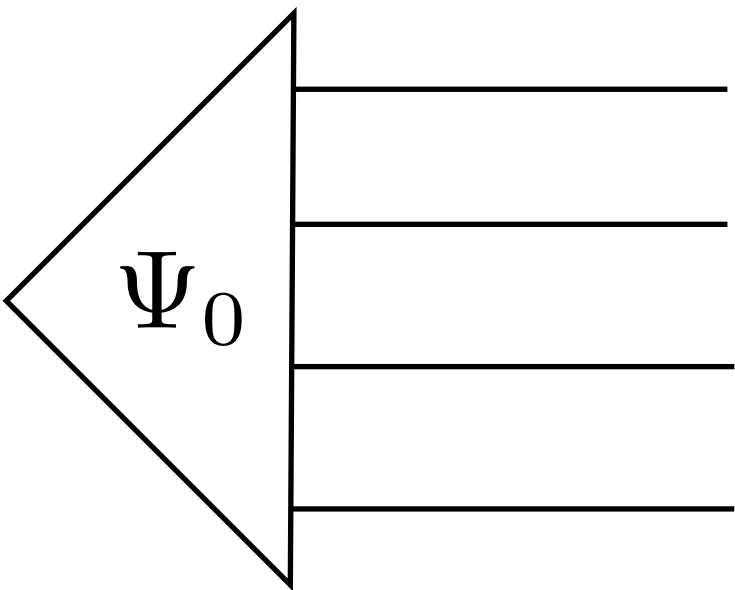
Goal: Retain detection capabilities on  $\mathbf{P}$  while differentiating between  $\mathbf{A}$  and  $\mathbf{P}$

Solution: Expand Hilbert space by adding ancilla qubits

# Asymmetric Code

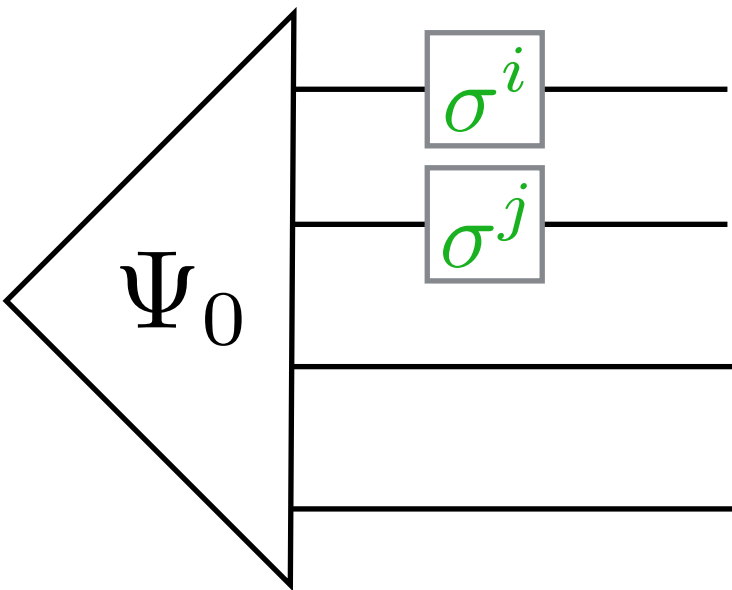
# Asymmetric Code

$$\mathcal{S}_0 = \langle XIXI, IXIX, ZIZI, IZIZ \rangle$$



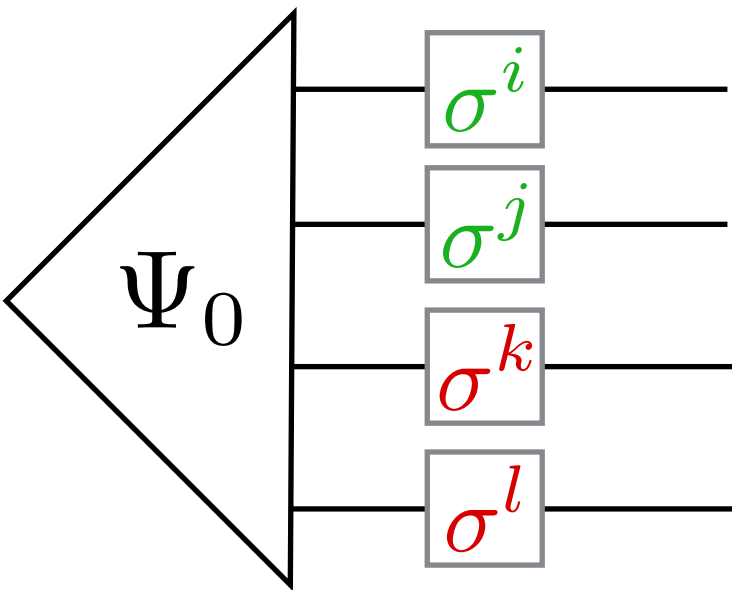
# Asymmetric Code

$$\mathcal{S}_0 = \langle XIXI, IXIX, ZIZI, IZIZ \rangle$$



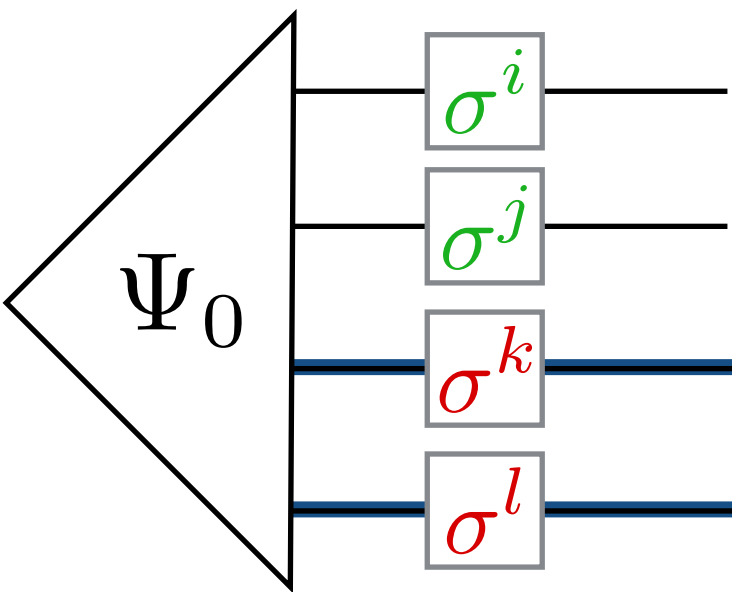
# Asymmetric Code

$$\mathcal{S}_0 = \langle XIXI, IXIX, ZIZI, IZIZ \rangle$$



# Asymmetric Code

$$\mathcal{S}_0 = \langle X I \bar{X} I, I X I \bar{X}, Z I \bar{Z} I, I Z I \bar{Z} \rangle$$



[[4,2,2]] Encoding

$$\mathcal{S}_4 = \langle X X X X, Z Z Z Z \rangle$$

$$\begin{array}{ll} \bar{X}_1 = X X I I & \bar{Z}_1 = Z I Z I \\ \bar{X}_2 = I X I X & \bar{Z}_2 = I I Z Z \end{array}$$

# Asymmetric Code

$$\mathcal{S}_0 = \langle X I \bar{X} I, I X I \bar{X}, Z I \bar{Z} I, I Z I \bar{Z} \rangle$$

$$\mathcal{S}_1 = \langle I I X X X X, I I Z Z Z Z, X I X X I I, \\ Z I Z I Z I, I X I X I X, I Z I I Z Z \rangle.$$

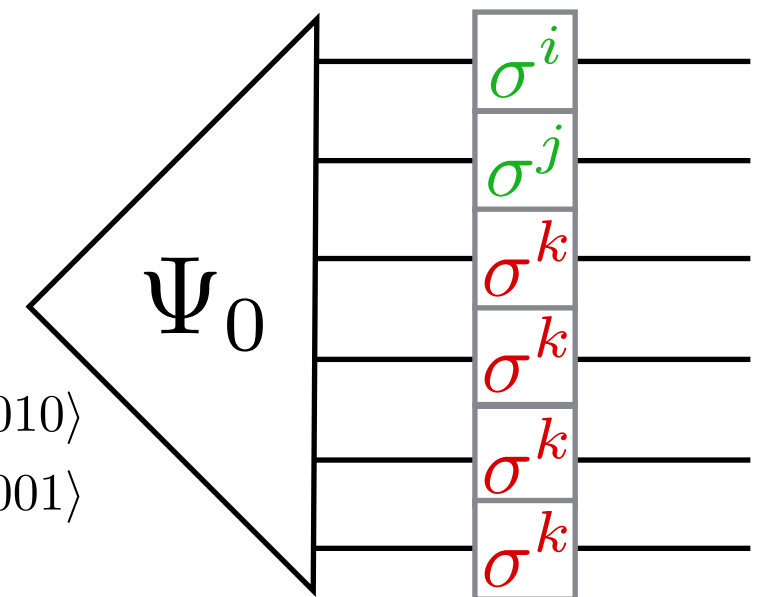
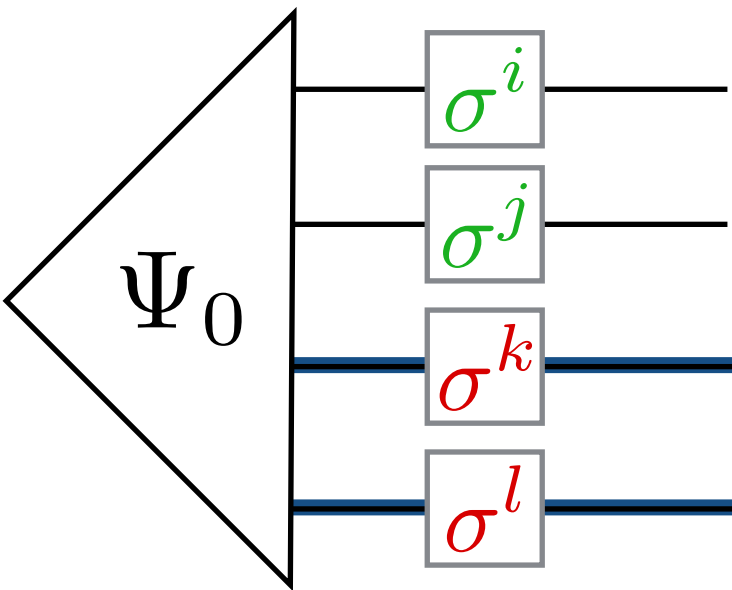
[[4,2,2]] Encoding

$$\mathcal{S}_4 = \langle X X X X, Z Z Z Z \rangle$$

$$\bar{X}_1 = X X I I \quad \bar{Z}_1 = Z I Z I$$

$$\bar{X}_2 = I X I X \quad \bar{Z}_2 = I I Z Z$$

$$|0\rangle = |000000\rangle + |001111\rangle + |010101\rangle + |011010\rangle \\ + |100011\rangle + |101100\rangle + |110110\rangle + |111001\rangle$$



# Asymmetric Code

$$\mathcal{S}_0 = \langle X I \bar{X} I, I X I \bar{X}, Z I \bar{Z} I, I Z I \bar{Z} \rangle$$

$$\mathcal{S}_1 = \langle I I X X X X, I I Z Z Z Z, X I X X I I, \\ Z I Z I Z I, I X I X I X, I Z I I Z Z \rangle.$$

[[4,2,2]] Encoding

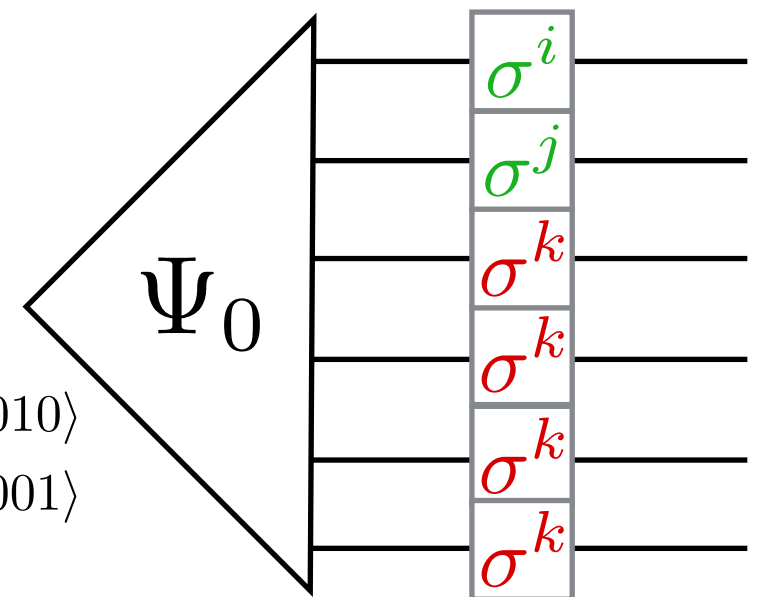
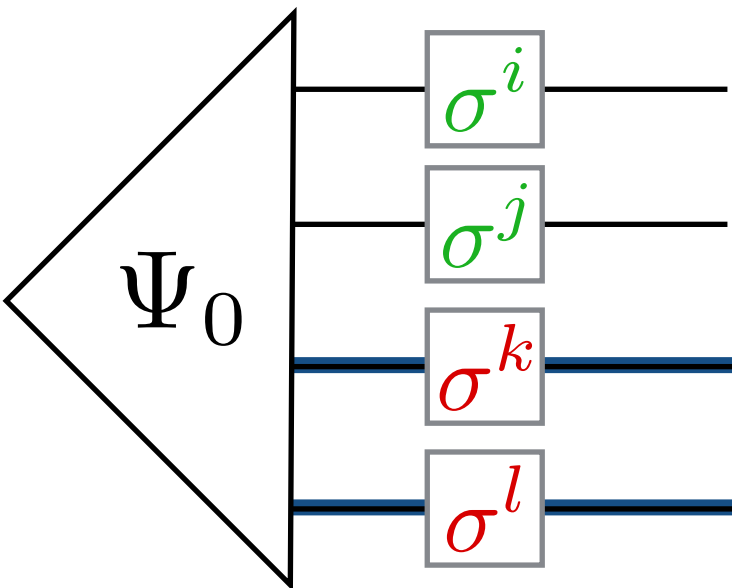
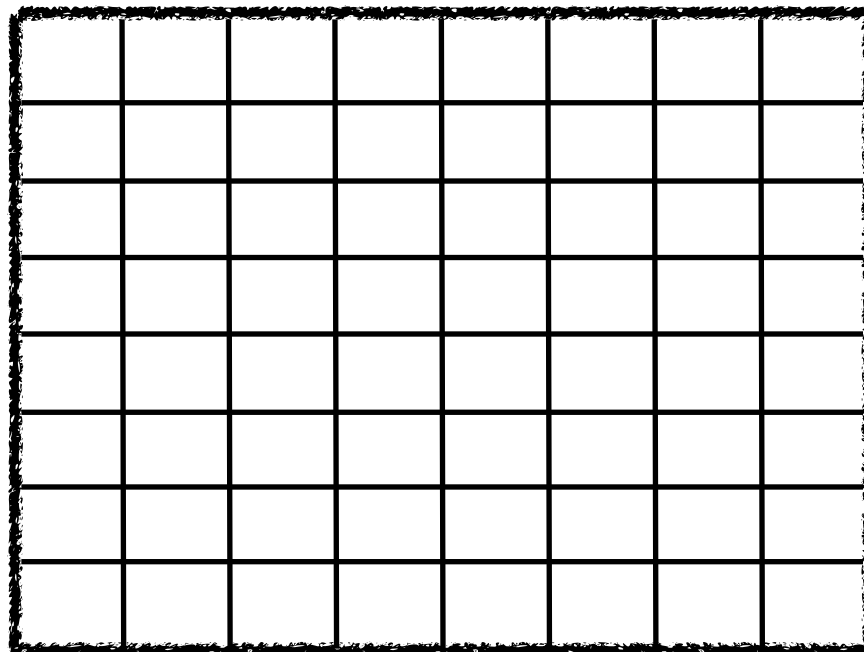
$$\mathcal{S}_4 = \langle X X X X, Z Z Z Z \rangle$$

$$\bar{X}_1 = X X I I \quad \bar{Z}_1 = Z I Z I$$

$$\bar{X}_2 = I X I X \quad \bar{Z}_2 = I I Z Z$$

$$|0\rangle = |000000\rangle + |001111\rangle + |010101\rangle + |011010\rangle \\ + |100011\rangle + |101100\rangle + |110110\rangle + |111001\rangle$$

II





# Asymmetric Code

$$\mathcal{S}_0 = \langle X I \overline{X} I, I X I \overline{X}, Z I \overline{Z} I, I Z I \overline{Z} \rangle$$

$$\mathcal{S}_1 = \langle IIXXXX, IIZZZZ, XIXXII, \\ ZIZIZI, IXIXIX, IZII ZZ \rangle.$$

## [[4,2,2]] Encoding

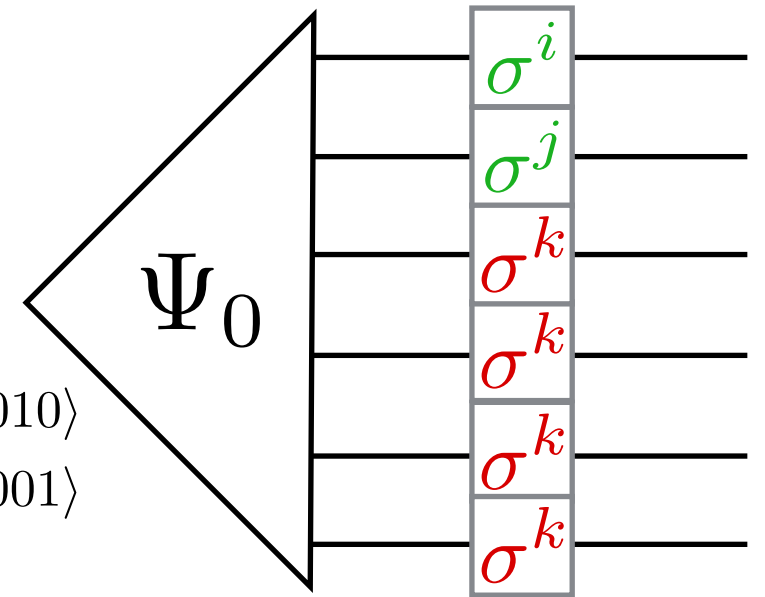
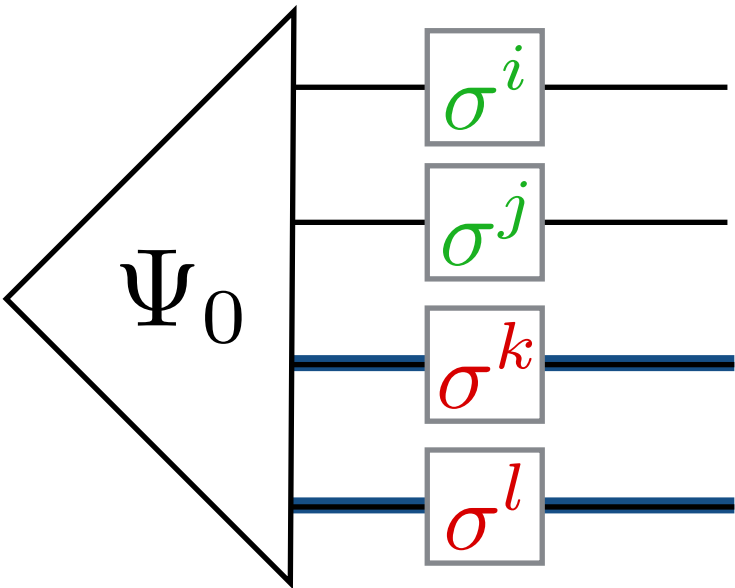
$$S_4 = \langle XXXX, ZZZZ \rangle$$

$$\overline{X}_1 = XXII \quad \overline{Z}_1 = ZIZI$$

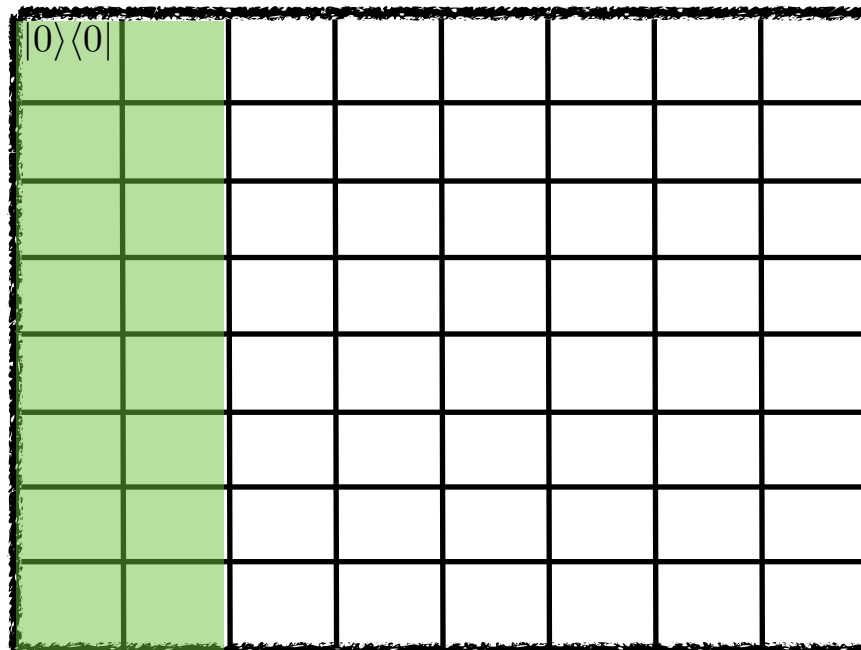
$$\overline{X}_2 = IXIX \quad \overline{Z}_2 = IIZZ$$

$$|0\rangle = |000000\rangle + |001111\rangle + |010101\rangle + |011010\rangle \\ + |100011\rangle + |101100\rangle + |110110\rangle + |111001\rangle$$

$$\sigma_1^i \sigma_2^j \otimes I^{\otimes 4}$$



II



# Asymmetric Code

$$\mathcal{S}_0 = \langle X I \bar{X} I, I X I \bar{X}, Z I \bar{Z} I, I Z I \bar{Z} \rangle$$

$$\mathcal{S}_1 = \langle I I X X X X, I I Z Z Z Z, X I X X I I, \\ Z I Z I Z I, I X I X I X, I Z I I Z Z \rangle.$$

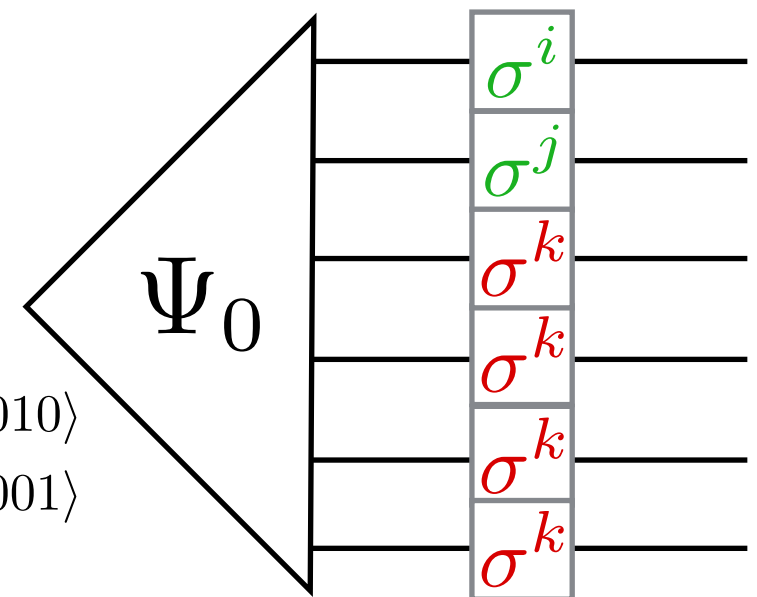
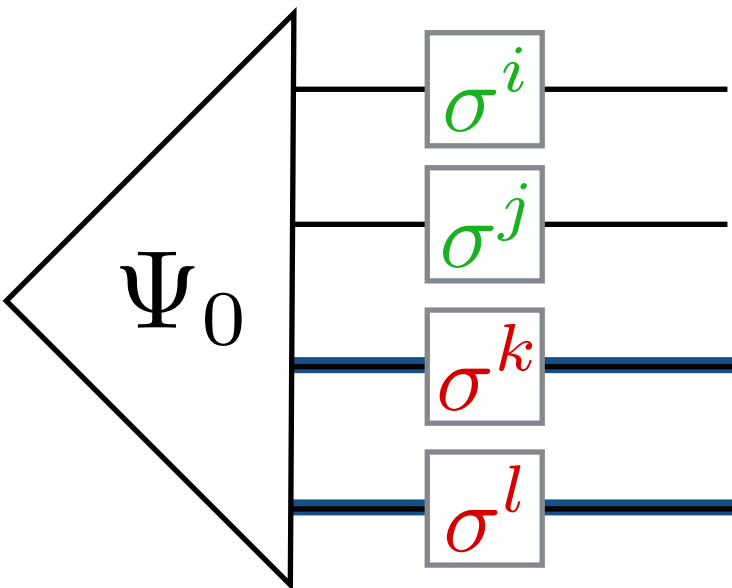
[[4,2,2]] Encoding

$$S_4 = \langle X X X X, Z Z Z Z \rangle$$

$$\bar{X}_1 = X X I I \quad \bar{Z}_1 = Z I Z I$$

$$\bar{X}_2 = I X I X \quad \bar{Z}_2 = I I Z Z$$

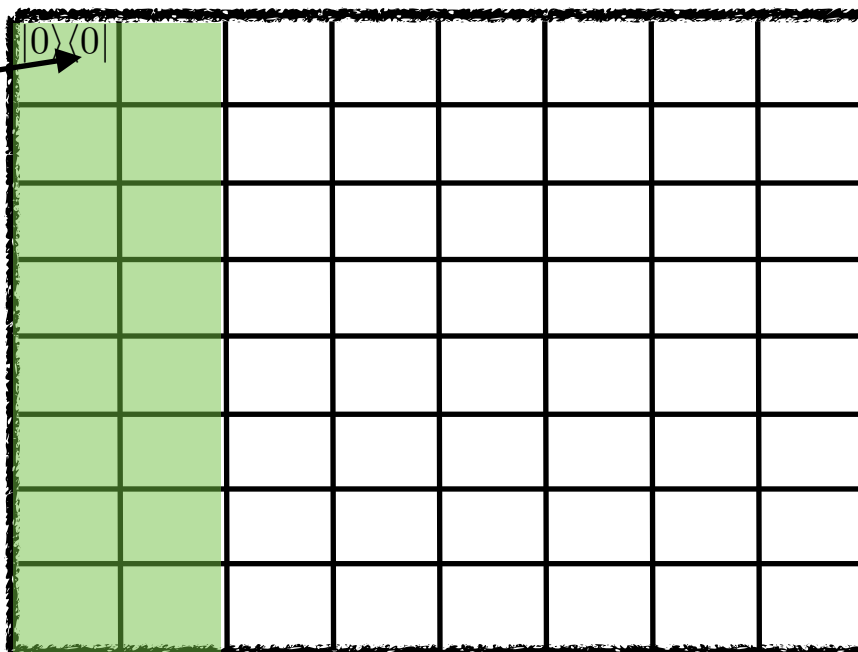
$$|0\rangle = |000000\rangle + |001111\rangle + |010101\rangle + |011010\rangle \\ + |100011\rangle + |101100\rangle + |110110\rangle + |111001\rangle$$



$$\sigma_1^i \sigma_2^j \otimes I^{\otimes 4}$$

II

I



# Asymmetric Code

$$\mathcal{S}_0 = \langle X I \bar{X} I, I X I \bar{X}, Z I \bar{Z} I, I Z I \bar{Z} \rangle$$

$$\mathcal{S}_1 = \langle I I X X X X, I I Z Z Z Z, X I X X I I, \\ Z I Z I Z I, I X I X I X, I Z I I Z Z \rangle.$$

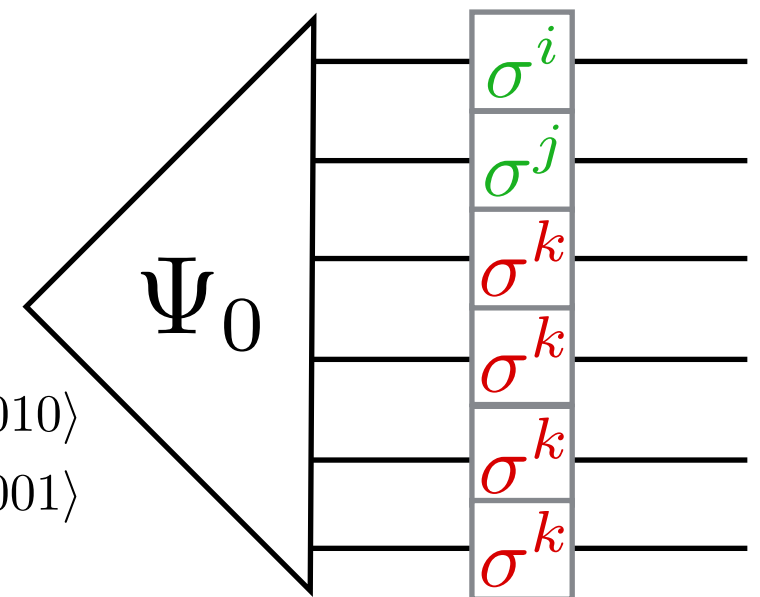
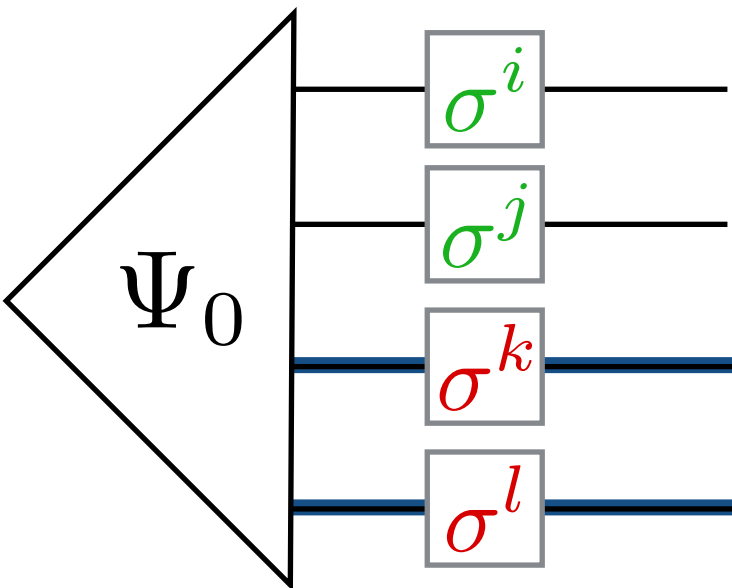
[[4,2,2]] Encoding

$$S_4 = \langle X X X X, Z Z Z Z \rangle$$

$$\bar{X}_1 = X X I I \quad \bar{Z}_1 = Z I Z I$$

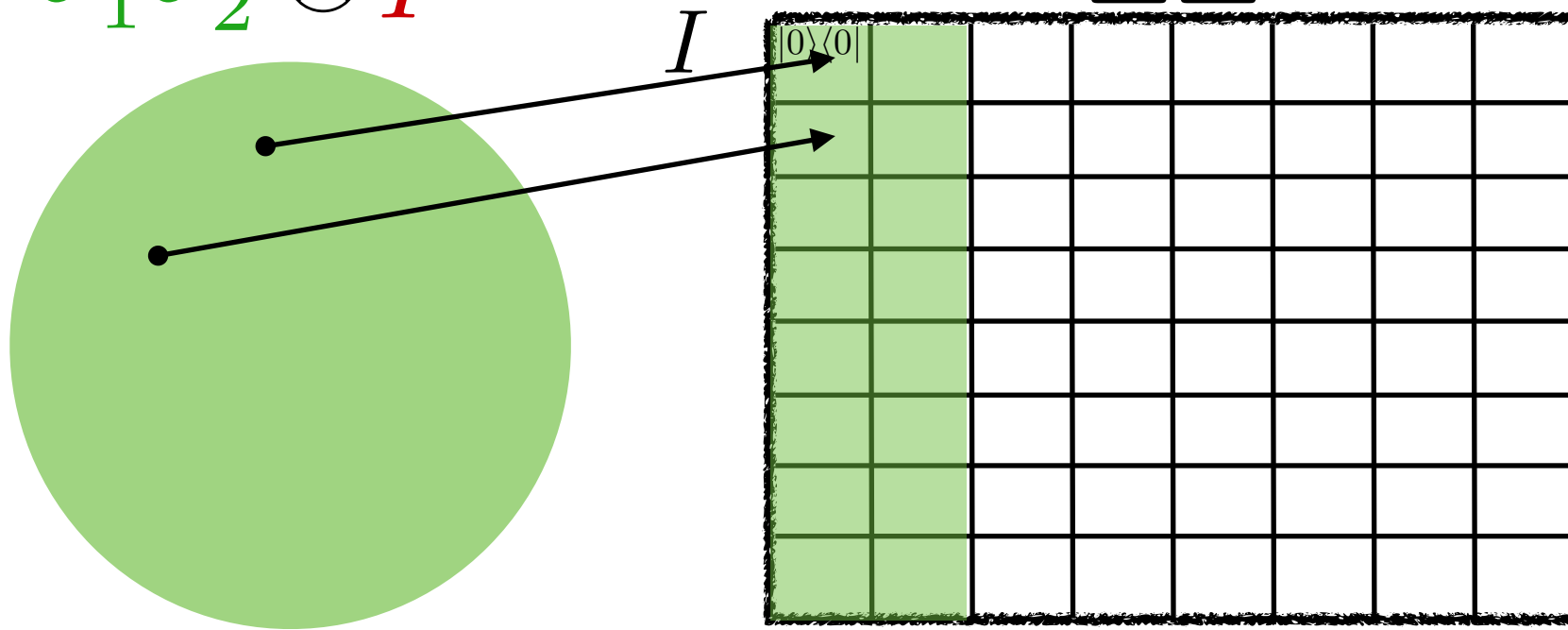
$$\bar{X}_2 = I X I X \quad \bar{Z}_2 = I I Z Z$$

$$|0\rangle = |000000\rangle + |001111\rangle + |010101\rangle + |011010\rangle \\ + |100011\rangle + |101100\rangle + |110110\rangle + |111001\rangle$$



$$\sigma_1^i \sigma_2^j \otimes I^{\otimes 4}$$

II



# Asymmetric Code

$$\mathcal{S}_0 = \langle X I \bar{X} I, I X I \bar{X}, Z I \bar{Z} I, I Z I \bar{Z} \rangle$$

$$\mathcal{S}_1 = \langle I I X X X X, I I Z Z Z Z, X I X X I I, \\ Z I Z I Z I, I X I X I X, I Z I I Z Z \rangle.$$

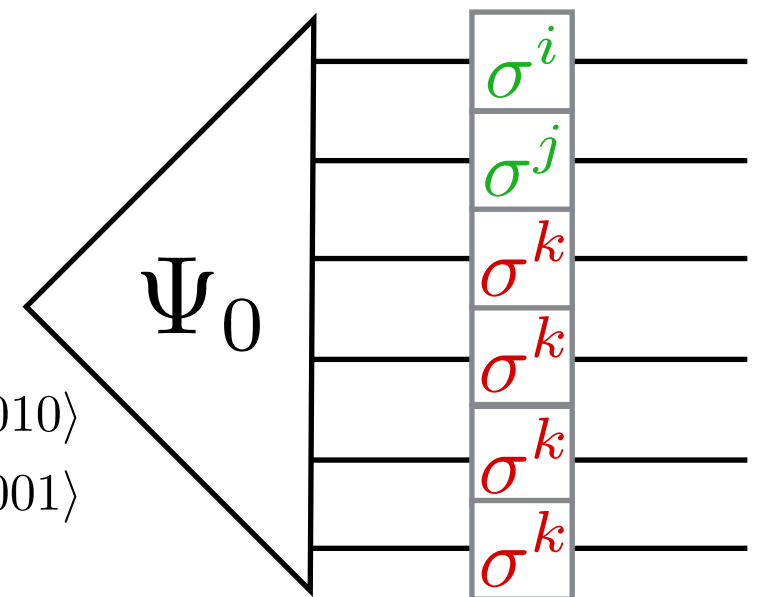
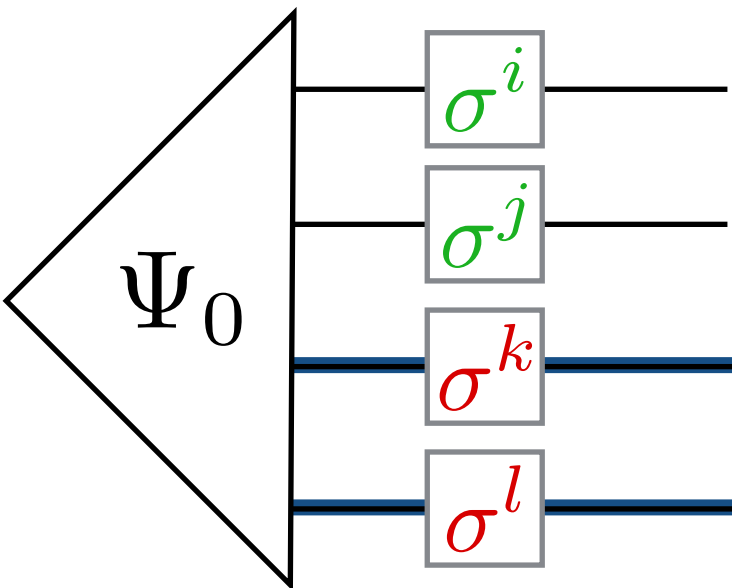
[[4,2,2]] Encoding

$$S_4 = \langle X X X X, Z Z Z Z \rangle$$

$$\bar{X}_1 = X X I I \quad \bar{Z}_1 = Z I Z I$$

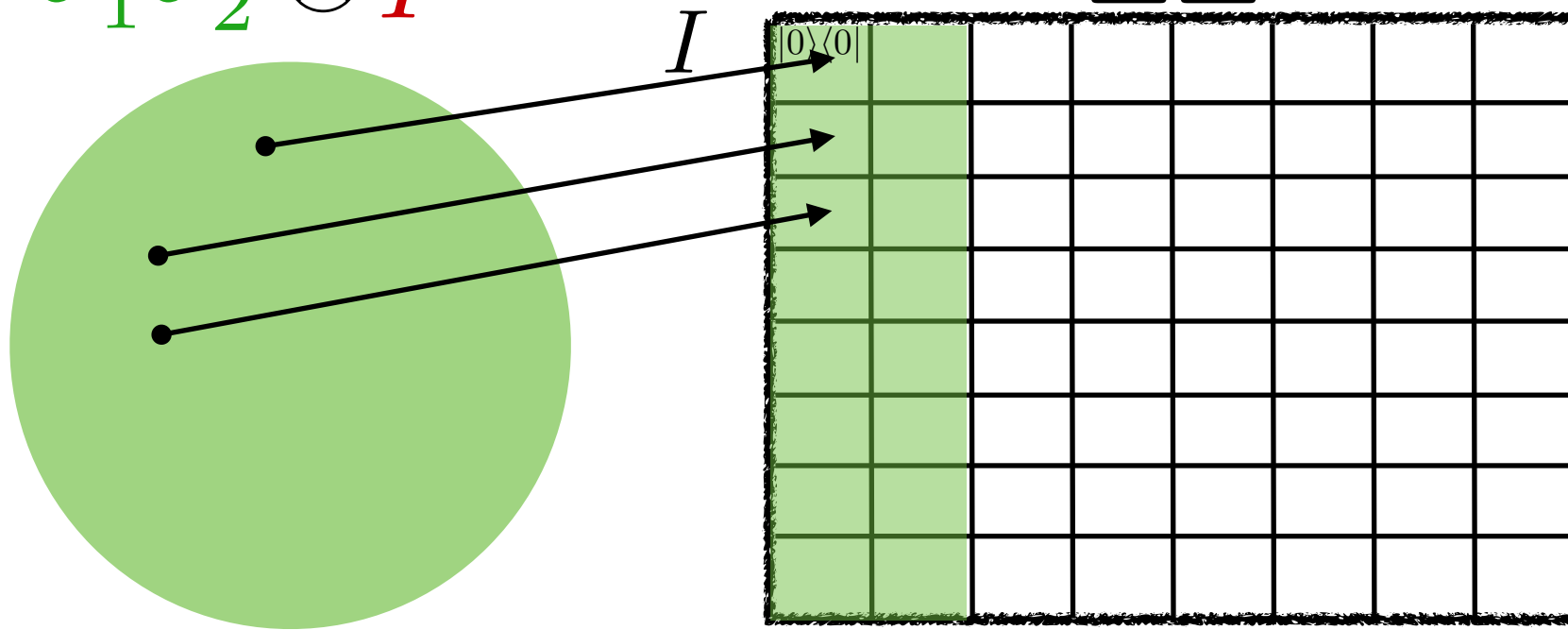
$$\bar{X}_2 = I X I X \quad \bar{Z}_2 = I I Z Z$$

$$|0\rangle = |000000\rangle + |001111\rangle + |010101\rangle + |011010\rangle \\ + |100011\rangle + |101100\rangle + |110110\rangle + |111001\rangle$$



$$\sigma_1^i \sigma_2^j \otimes I^{\otimes 4}$$

II



# Asymmetric Code

$$\mathcal{S}_0 = \langle X I \bar{X} I, I X I \bar{X}, Z I \bar{Z} I, I Z I \bar{Z} \rangle$$

$$\mathcal{S}_1 = \langle I I X X X X, I I Z Z Z Z, X I X X I I, \\ Z I Z I Z I, I X I X I X, I Z I I Z Z \rangle.$$

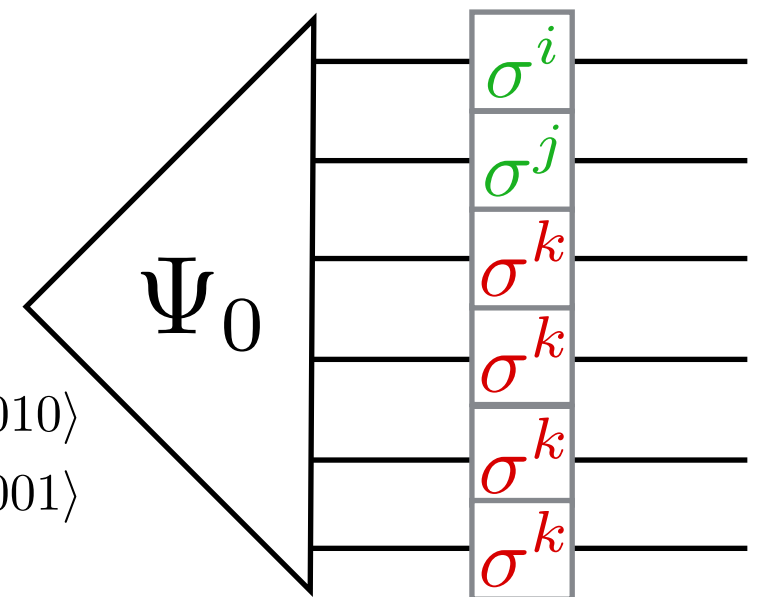
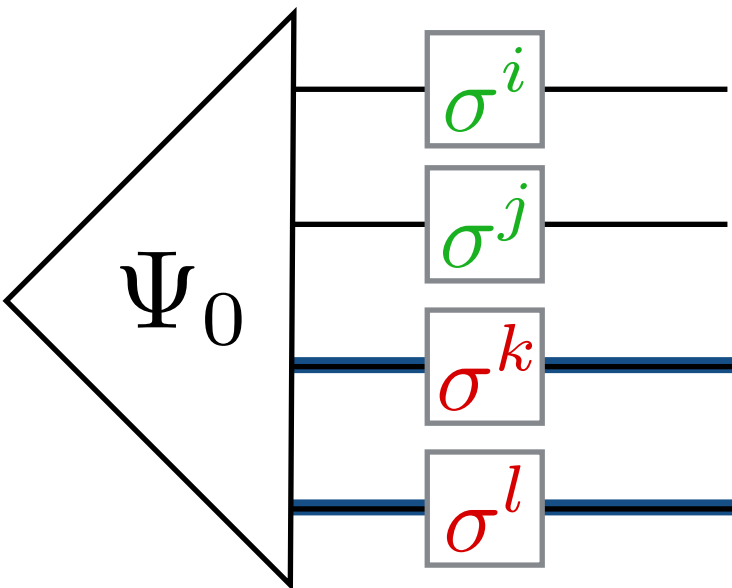
[[4,2,2]] Encoding

$$S_4 = \langle X X X X, Z Z Z Z \rangle$$

$$\bar{X}_1 = X X I I \quad \bar{Z}_1 = Z I Z I$$

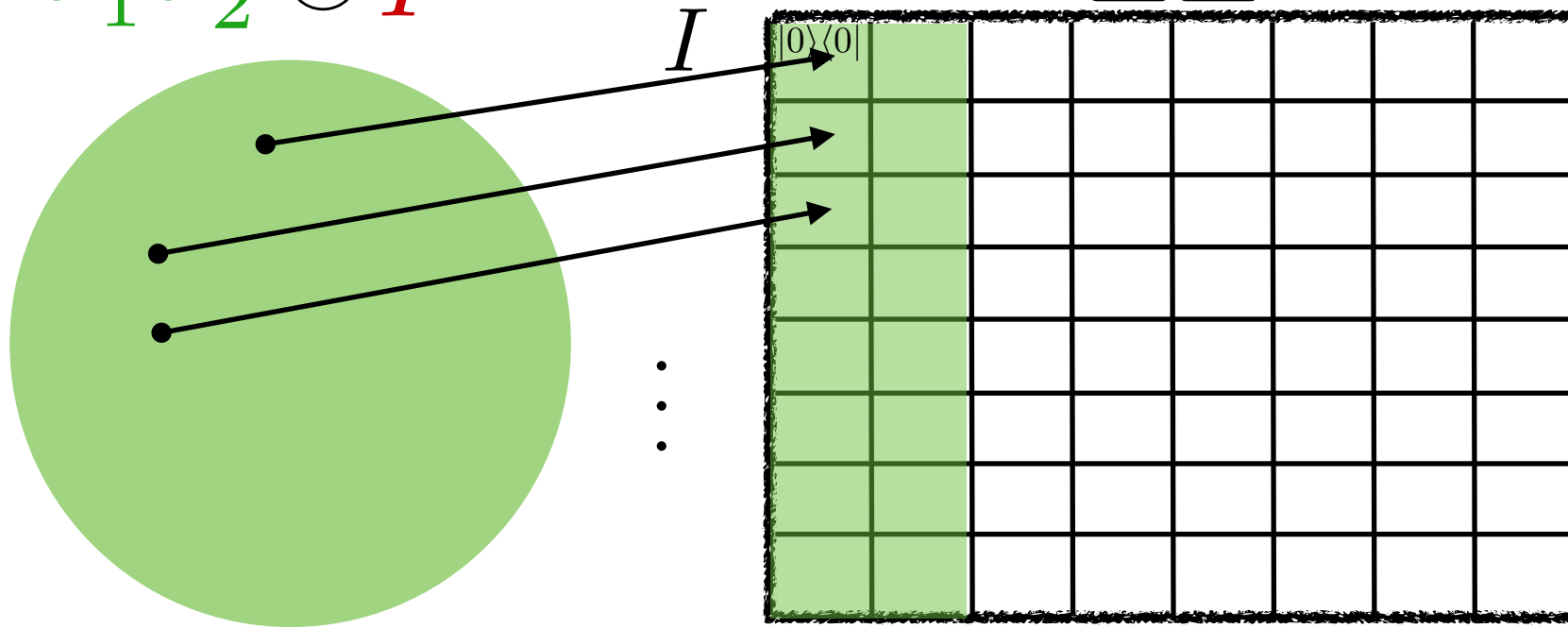
$$\bar{X}_2 = I X I X \quad \bar{Z}_2 = I I Z Z$$

$$|0\rangle = |000000\rangle + |001111\rangle + |010101\rangle + |011010\rangle \\ + |100011\rangle + |101100\rangle + |110110\rangle + |111001\rangle$$



$$\sigma_1^i \sigma_2^j \otimes I^{\otimes 4}$$

II



# Asymmetric Code

$$\mathcal{S}_0 = \langle X I \bar{X} I, I X I \bar{X}, Z I \bar{Z} I, I Z I \bar{Z} \rangle$$

$$\mathcal{S}_1 = \langle I I X X X X, I I Z Z Z Z, X I X X I I, \\ Z I Z I Z I, I X I X I X, I Z I I Z Z \rangle.$$

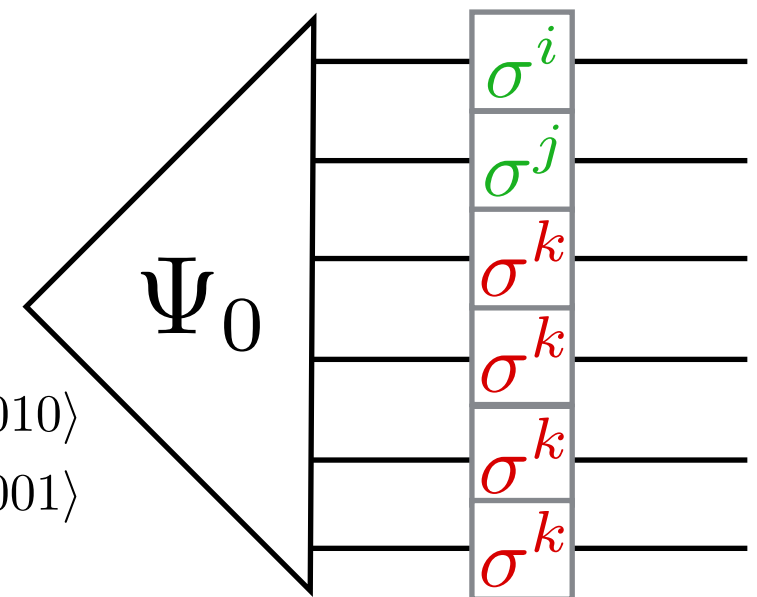
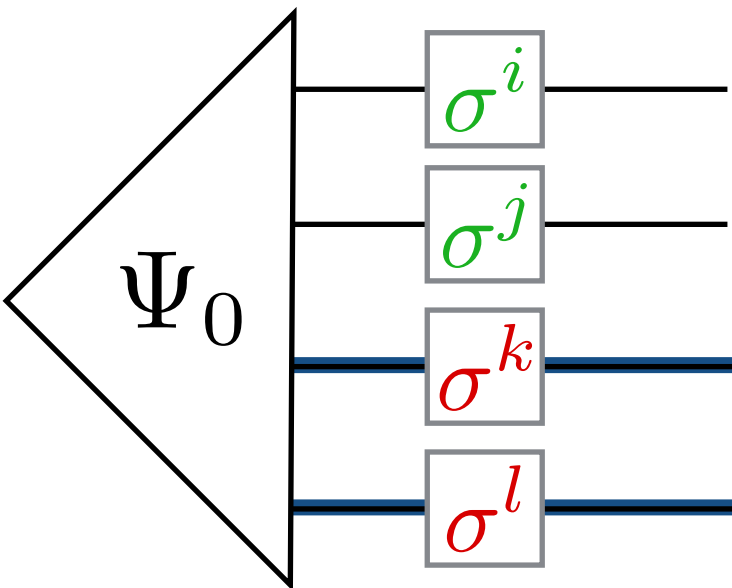
[[4,2,2]] Encoding

$$S_4 = \langle X X X X, Z Z Z Z \rangle$$

$$\bar{X}_1 = X X I I \quad \bar{Z}_1 = Z I Z I$$

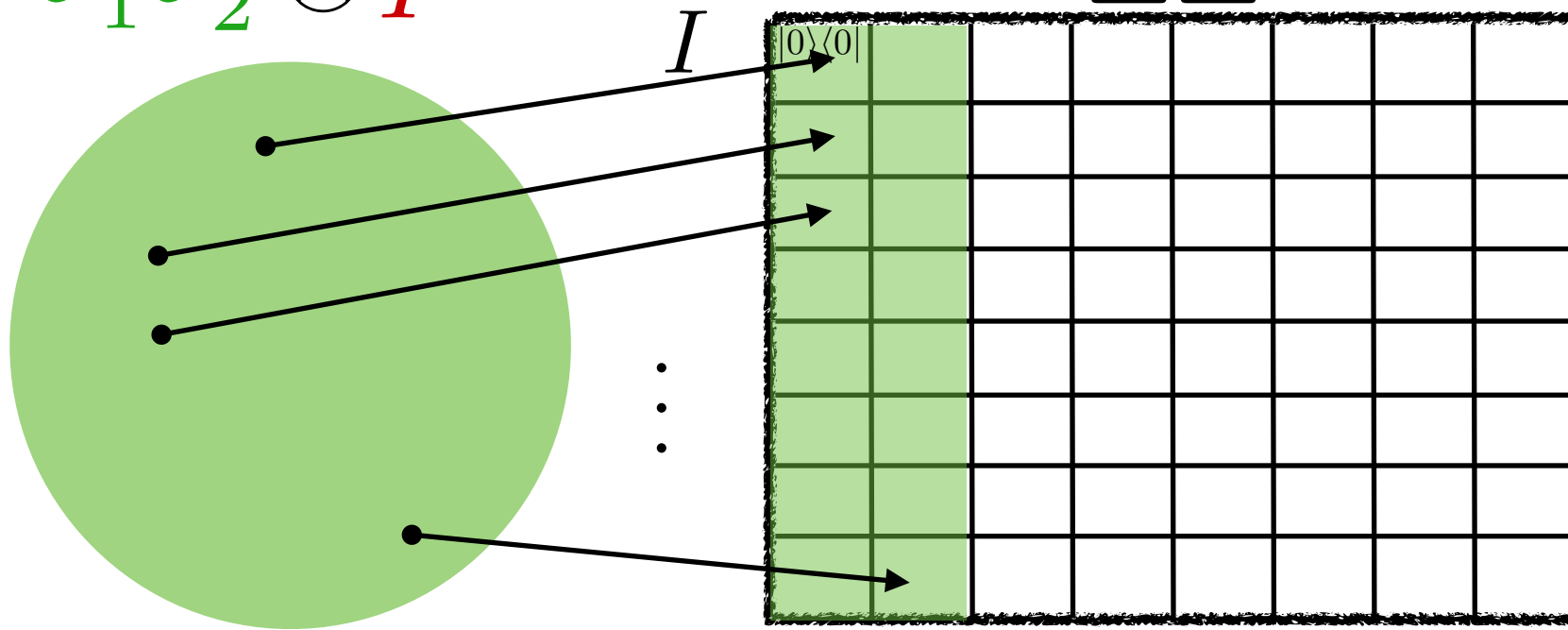
$$\bar{X}_2 = I X I X \quad \bar{Z}_2 = I I Z Z$$

$$|0\rangle = |000000\rangle + |001111\rangle + |010101\rangle + |011010\rangle \\ + |100011\rangle + |101100\rangle + |110110\rangle + |111001\rangle$$



$$\sigma_1^i \sigma_2^j \otimes I^{\otimes 4}$$

II



# Asymmetric Code

$$\mathcal{S}_0 = \langle X I \bar{X} I, I X I \bar{X}, Z I \bar{Z} I, I Z I \bar{Z} \rangle$$

$$\mathcal{S}_1 = \langle I I X X X X, I I Z Z Z Z, X I X X I I, Z I Z I Z I, I X I X I X, I Z I I Z Z \rangle.$$

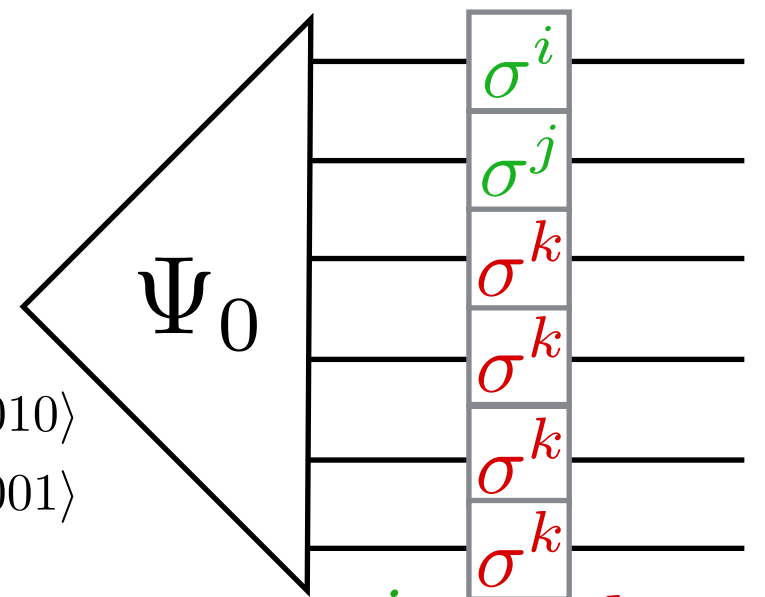
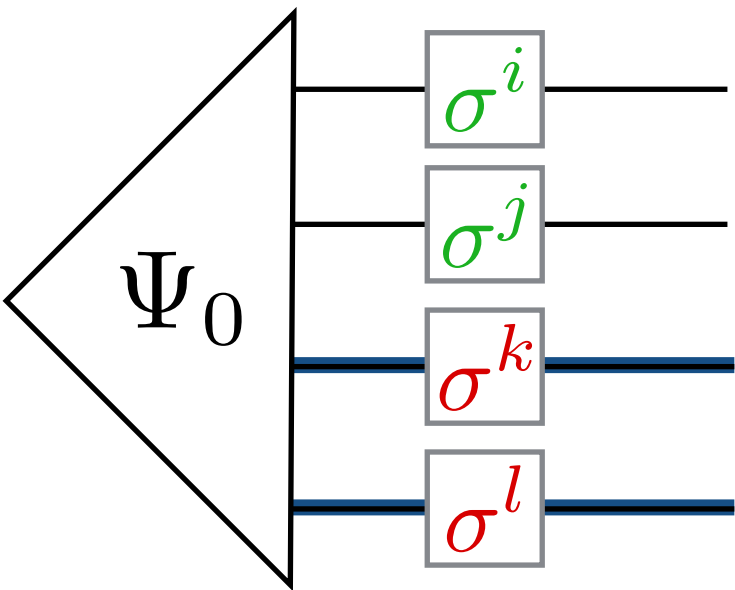
[[4,2,2]] Encoding

$$S_4 = \langle X X X X, Z Z Z Z \rangle$$

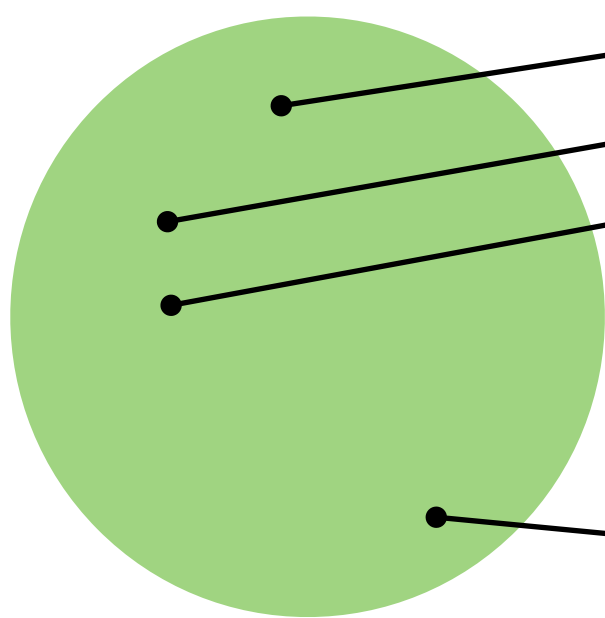
$$\bar{X}_1 = X X I I \quad \bar{Z}_1 = Z I Z I$$

$$\bar{X}_2 = I X I X \quad \bar{Z}_2 = I I Z Z$$

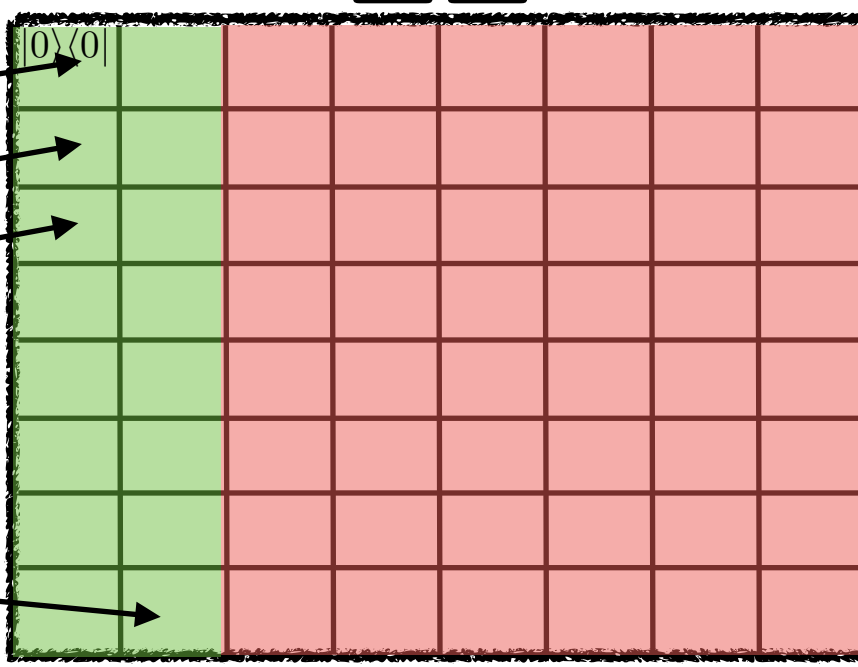
$$|0\rangle = |000000\rangle + |001111\rangle + |010101\rangle + |011010\rangle \\ + |100011\rangle + |101100\rangle + |110110\rangle + |111001\rangle$$



$$\sigma_1^i \sigma_2^j \otimes I^{\otimes 4}$$

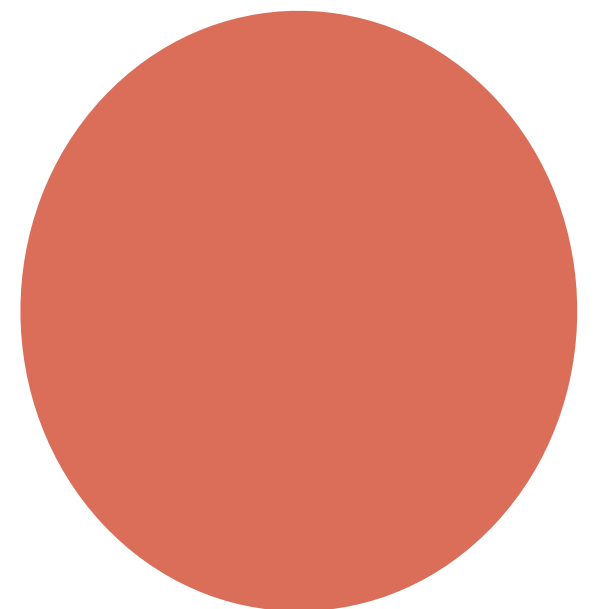


$I$



$H$

$$\sigma_1^i \sigma_2^j \otimes \sigma_z^k \otimes I^{\otimes 3}$$



# Asymmetric Code

$$\mathcal{S}_0 = \langle X I \bar{X} I, I X I \bar{X}, Z I \bar{Z} I, I Z I \bar{Z} \rangle$$

$$\mathcal{S}_1 = \langle I I X X X X, I I Z Z Z Z, X I X X I I, Z I Z I Z I, I X I X I X, I Z I I Z Z \rangle.$$

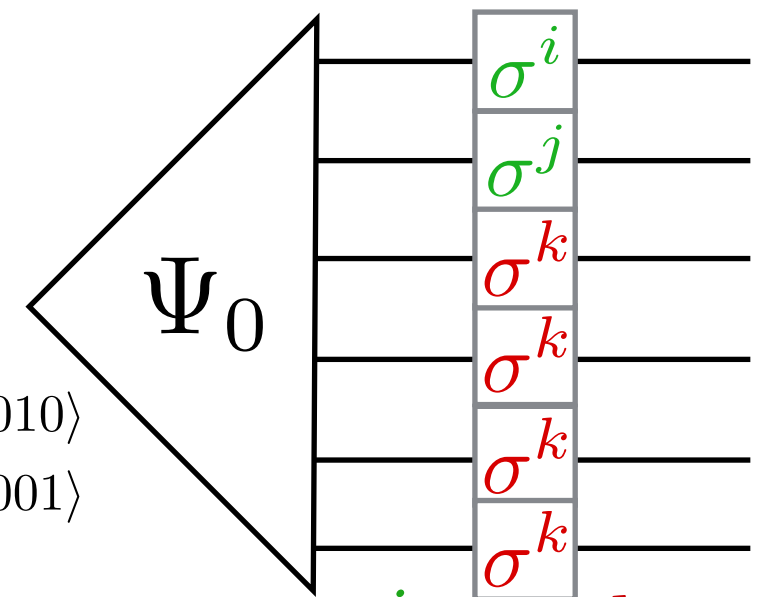
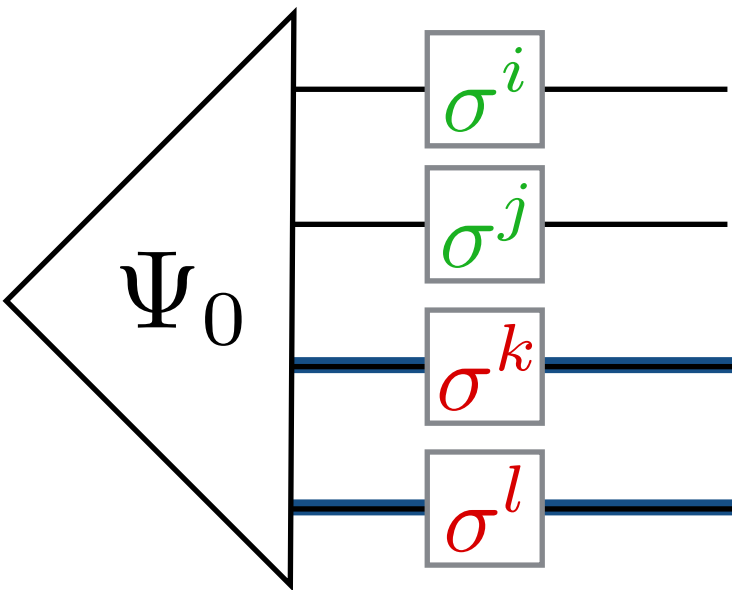
[[4,2,2]] Encoding

$$S_4 = \langle X X X X, Z Z Z Z \rangle$$

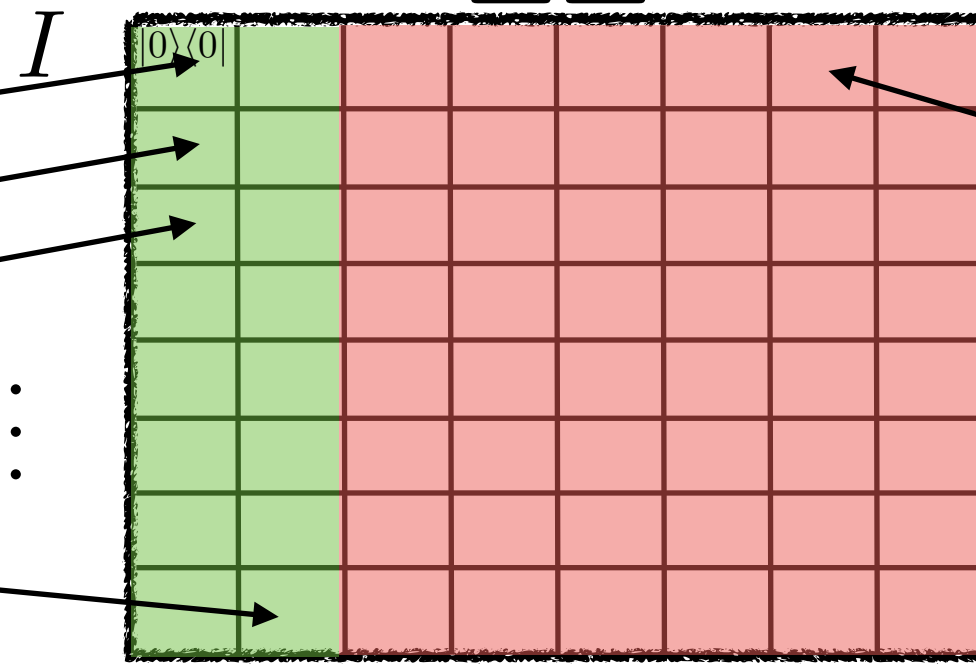
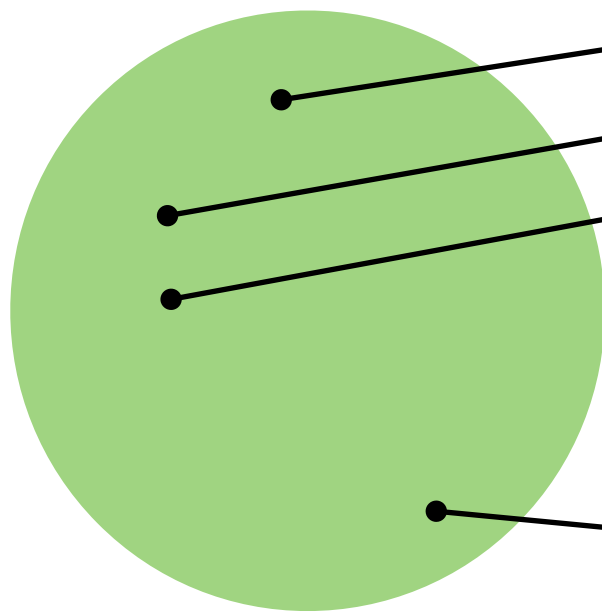
$$\bar{X}_1 = X X I I \quad \bar{Z}_1 = Z I Z I$$

$$\bar{X}_2 = I X I X \quad \bar{Z}_2 = I I Z Z$$

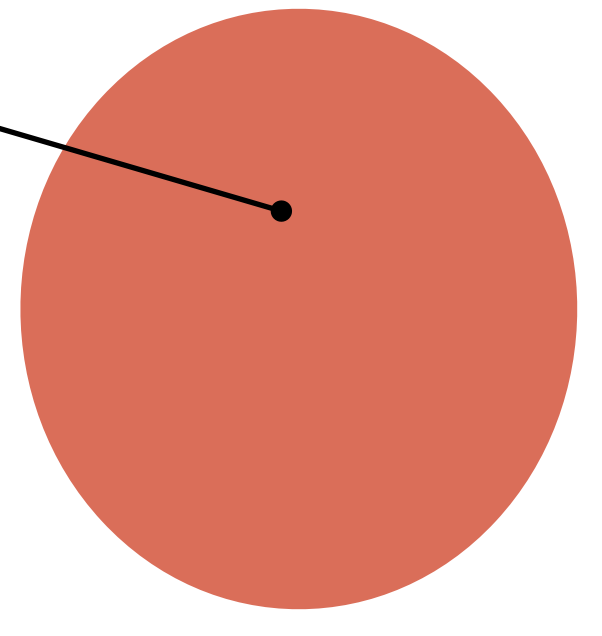
$$|0\rangle = |000000\rangle + |001111\rangle + |010101\rangle + |011010\rangle \\ + |100011\rangle + |101100\rangle + |110110\rangle + |111001\rangle$$



$$\sigma_1^i \sigma_2^j \otimes I^{\otimes 4}$$



$$\sigma_1^i \sigma_2^j \otimes \sigma_z^k \otimes I^{\otimes 3}$$





# Asymmetric Code

$$\mathcal{S}_0 = \langle X I \bar{X} I, I X I \bar{X}, Z I \bar{Z} I, I Z I \bar{Z} \rangle$$

$$\mathcal{S}_1 = \langle I I X X X X, I I Z Z Z Z, X I X X I I, Z I Z I Z I, I X I X I X, I Z I I Z Z \rangle.$$

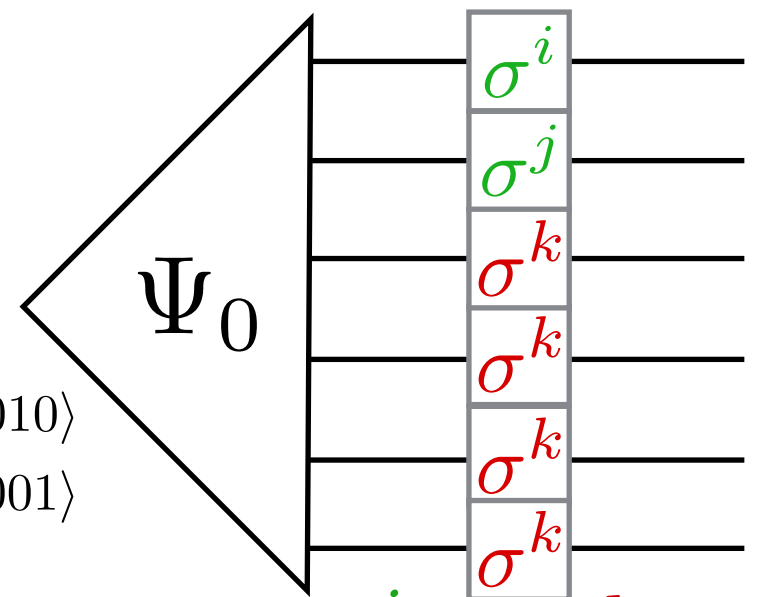
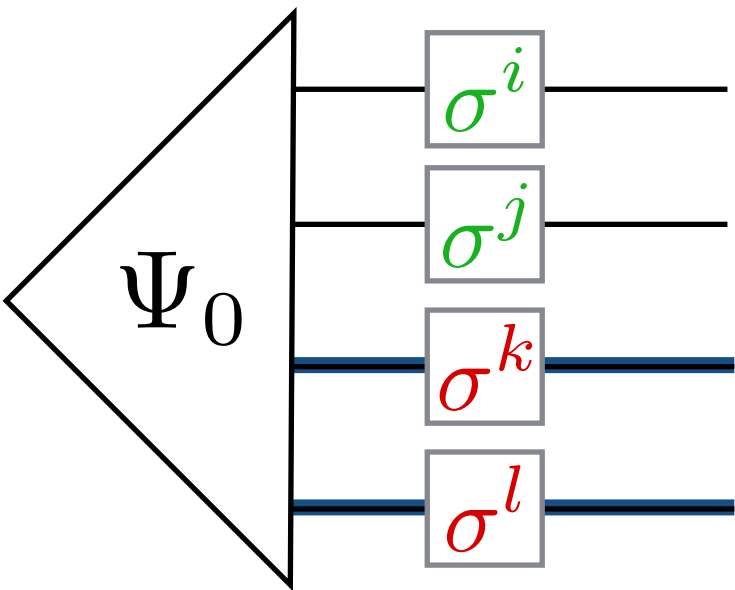
[[4,2,2]] Encoding

$$S_4 = \langle X X X X, Z Z Z Z \rangle$$

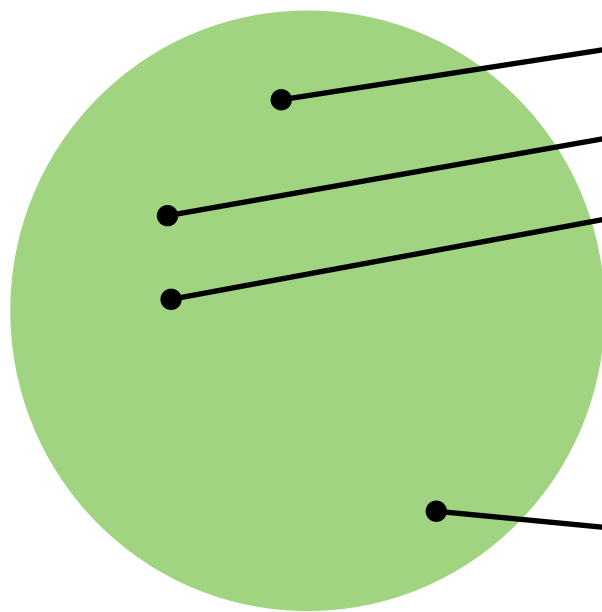
$$\bar{X}_1 = X X I I \quad \bar{Z}_1 = Z I Z I$$

$$\bar{X}_2 = I X I X \quad \bar{Z}_2 = I I Z Z$$

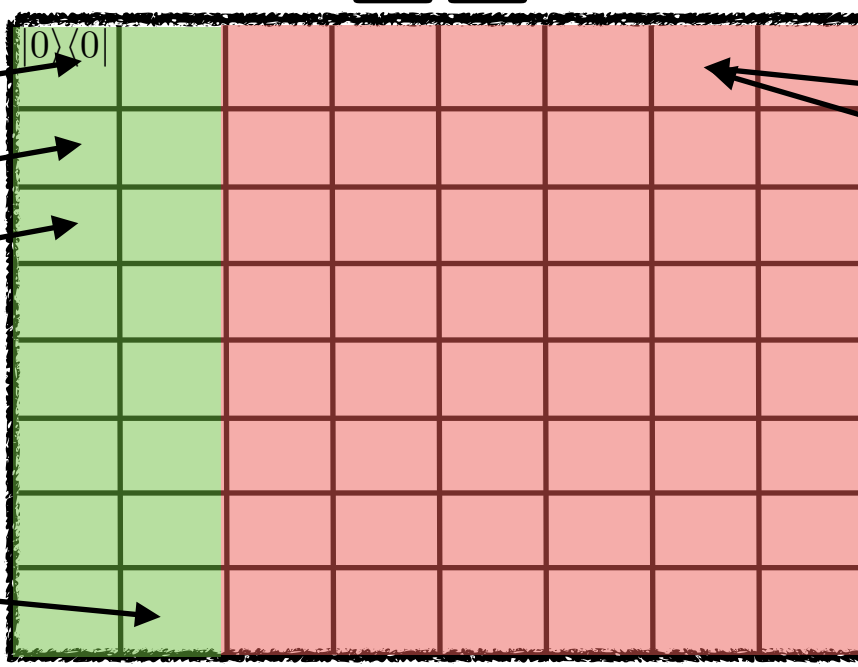
$$|0\rangle = |000000\rangle + |001111\rangle + |010101\rangle + |011010\rangle \\ + |100011\rangle + |101100\rangle + |110110\rangle + |111001\rangle$$



$$\sigma_1^i \sigma_2^j \otimes I^{\otimes 4}$$

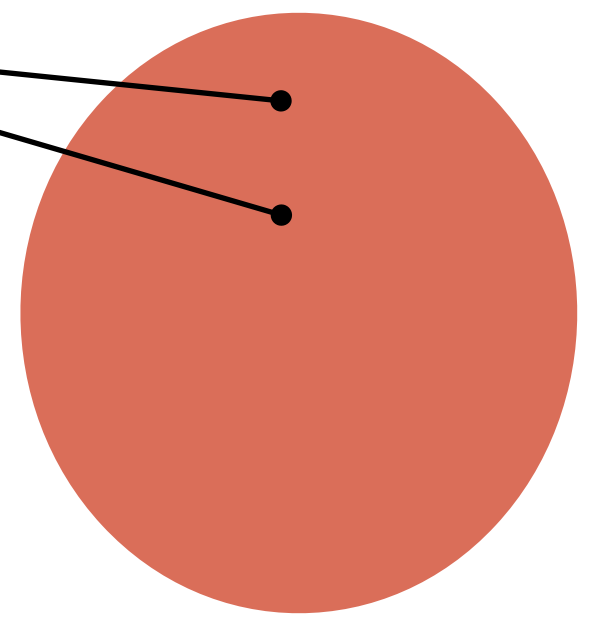


$I$



$H$

$$\sigma_1^i \sigma_2^j \otimes \sigma_z^k \otimes I^{\otimes 3}$$



# Asymmetric Code

$$\mathcal{S}_0 = \langle X I \bar{X} I, I X I \bar{X}, Z I \bar{Z} I, I Z I \bar{Z} \rangle$$

$$\mathcal{S}_1 = \langle I I X X X X, I I Z Z Z Z, X I X X I I, Z I Z I Z I, I X I X I X, I Z I I Z Z \rangle.$$

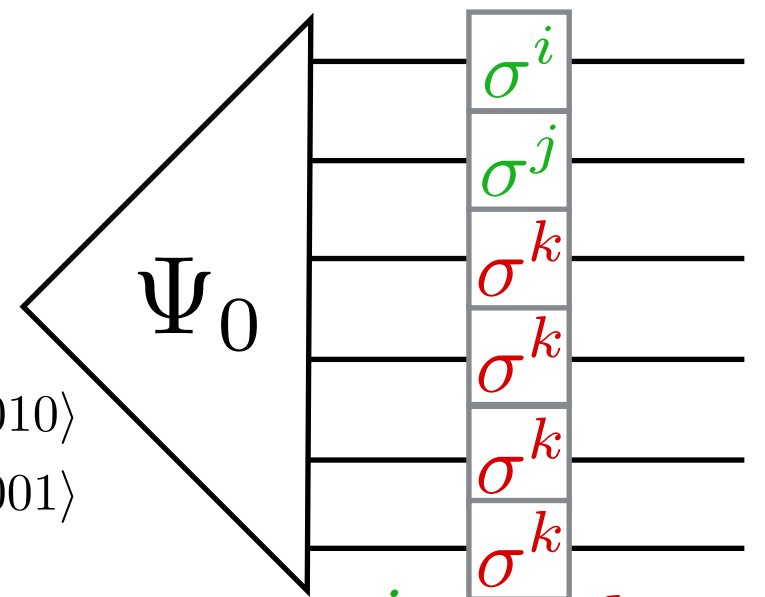
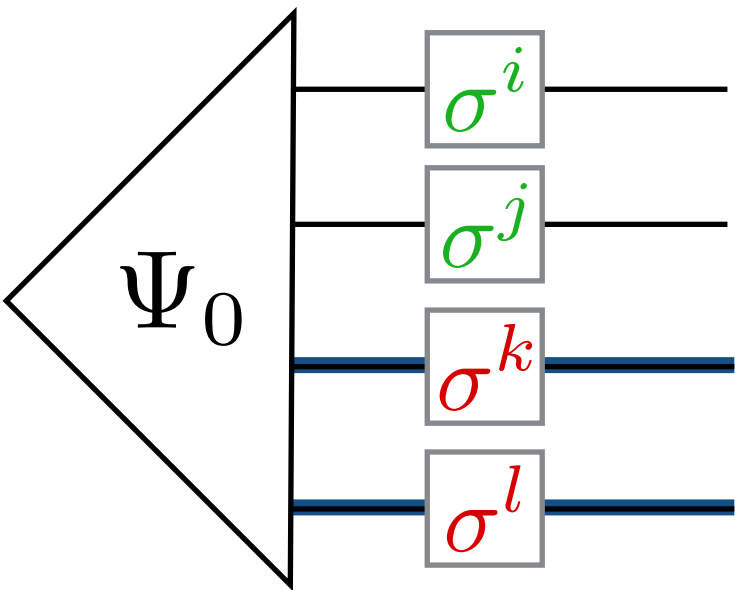
[[4,2,2]] Encoding

$$S_4 = \langle X X X X, Z Z Z Z \rangle$$

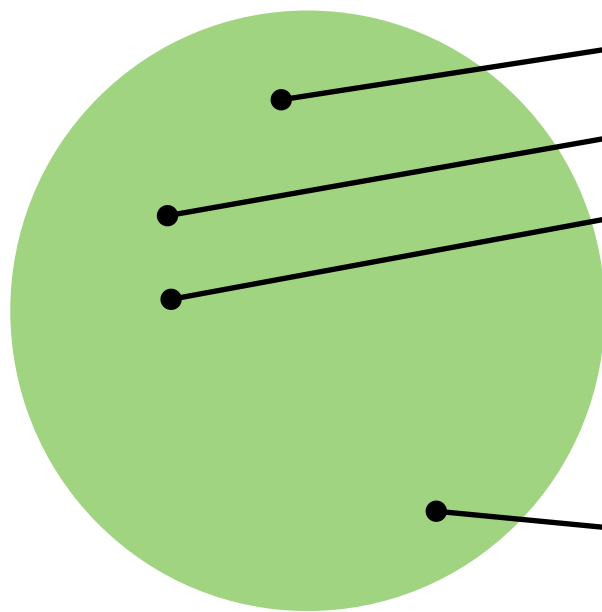
$$\bar{X}_1 = X X I I \quad \bar{Z}_1 = Z I Z I$$

$$\bar{X}_2 = I X I X \quad \bar{Z}_2 = I I Z Z$$

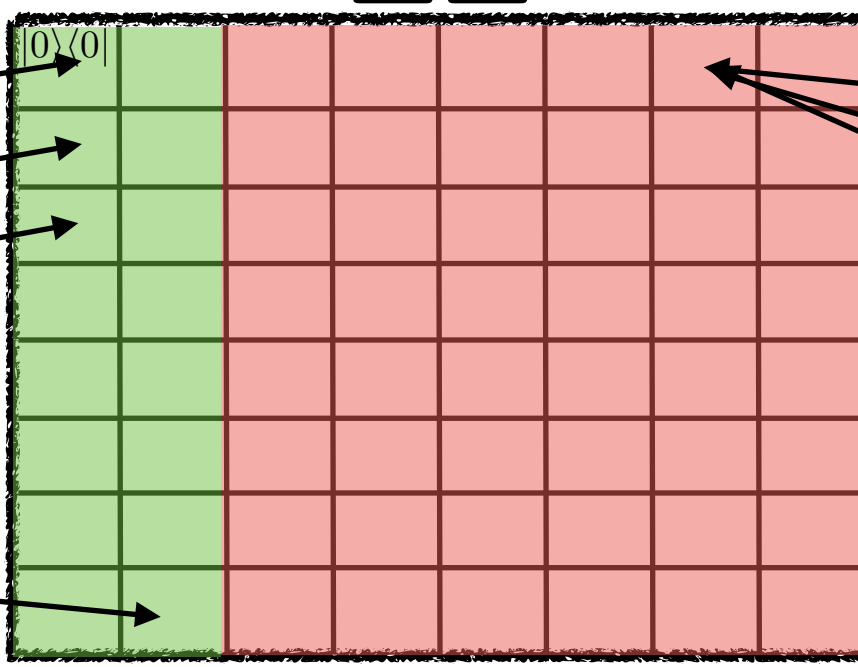
$$|0\rangle = |000000\rangle + |001111\rangle + |010101\rangle + |011010\rangle \\ + |100011\rangle + |101100\rangle + |110110\rangle + |111001\rangle$$



$$\sigma_1^i \sigma_2^j \otimes I^{\otimes 4}$$

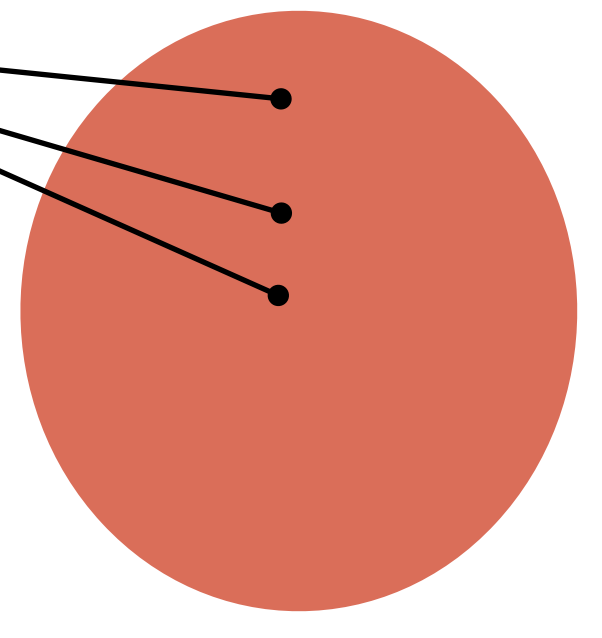


$I$



$H$

$$\sigma_1^i \sigma_2^j \otimes \sigma_z^k \otimes I^{\otimes 3}$$



# Asymmetric Code

$$\mathcal{S}_0 = \langle X I \bar{X} I, I X I \bar{X}, Z I \bar{Z} I, I Z I \bar{Z} \rangle$$

$$\mathcal{S}_1 = \langle I I X X X X, I I Z Z Z Z, X I X X I I, Z I Z I Z I, I X I X I X, I Z I I Z Z \rangle.$$

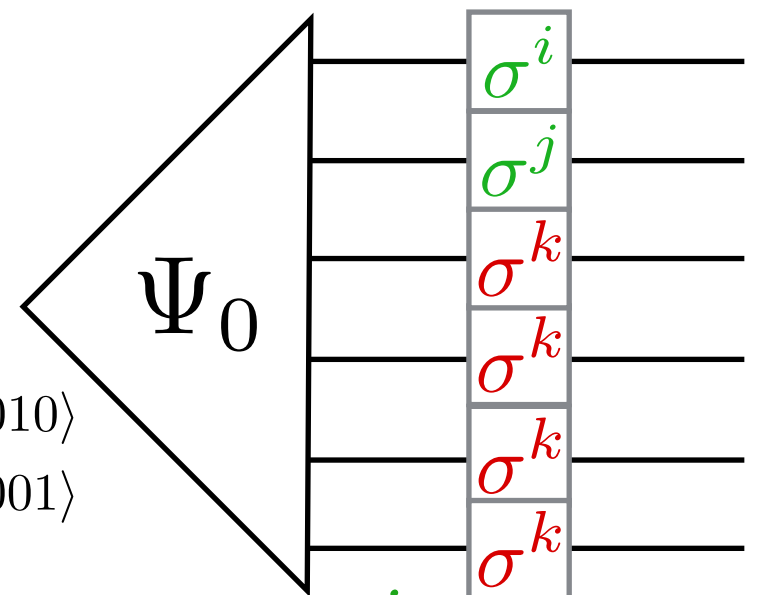
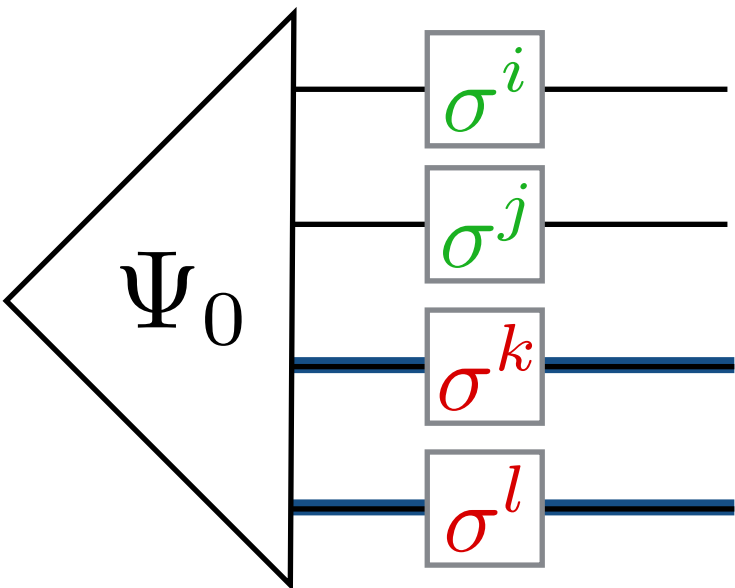
[[4,2,2]] Encoding

$$S_4 = \langle X X X X, Z Z Z Z \rangle$$

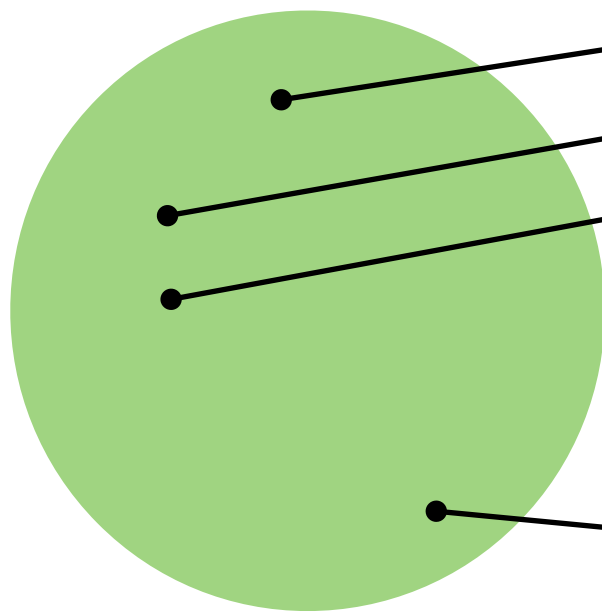
$$\bar{X}_1 = X X I I \quad \bar{Z}_1 = Z I Z I$$

$$\bar{X}_2 = I X I X \quad \bar{Z}_2 = I I Z Z$$

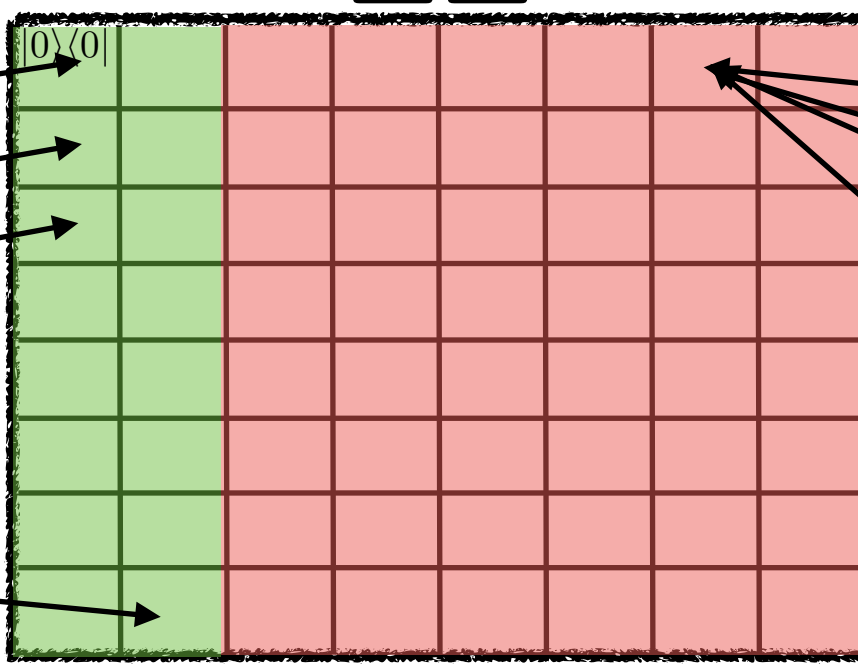
$$|0\rangle = |000000\rangle + |001111\rangle + |010101\rangle + |011010\rangle \\ + |100011\rangle + |101100\rangle + |110110\rangle + |111001\rangle$$



$$\sigma_1^i \sigma_2^j \otimes I^{\otimes 4}$$

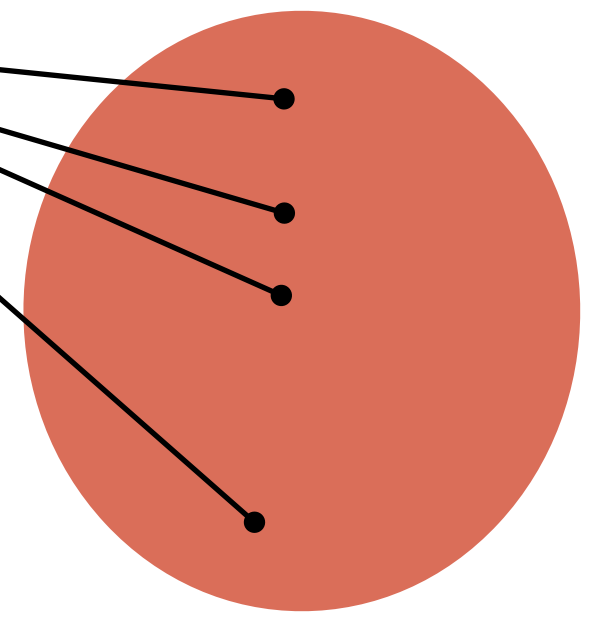


$I$



$H$

$$\sigma_1^i \sigma_2^j \otimes \sigma_z^k \otimes I^{\otimes 3}$$



# Asymmetric Code

$$\mathcal{S}_0 = \langle X I \bar{X} I, I X I \bar{X}, Z I \bar{Z} I, I Z I \bar{Z} \rangle$$

$$\mathcal{S}_1 = \langle I I X X X X, I I Z Z Z Z, X I X X I I, Z I Z I Z I, I X I X I X, I Z I I Z Z \rangle.$$

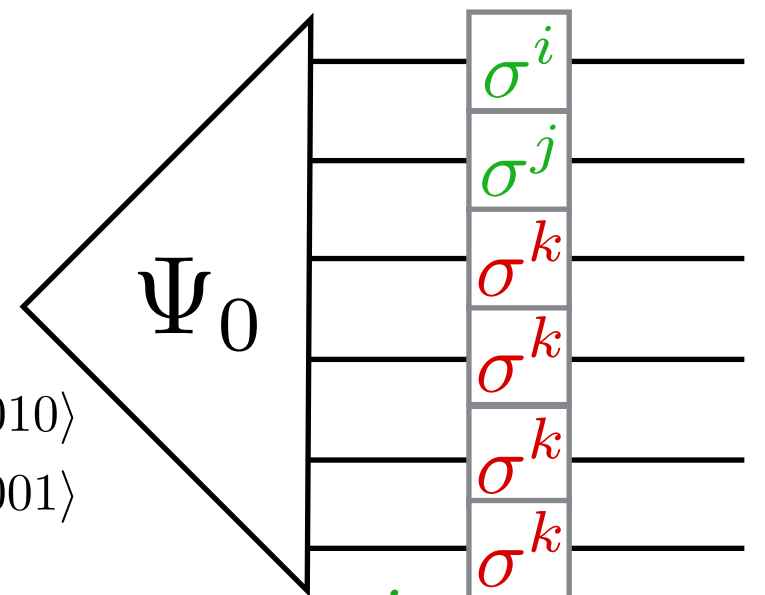
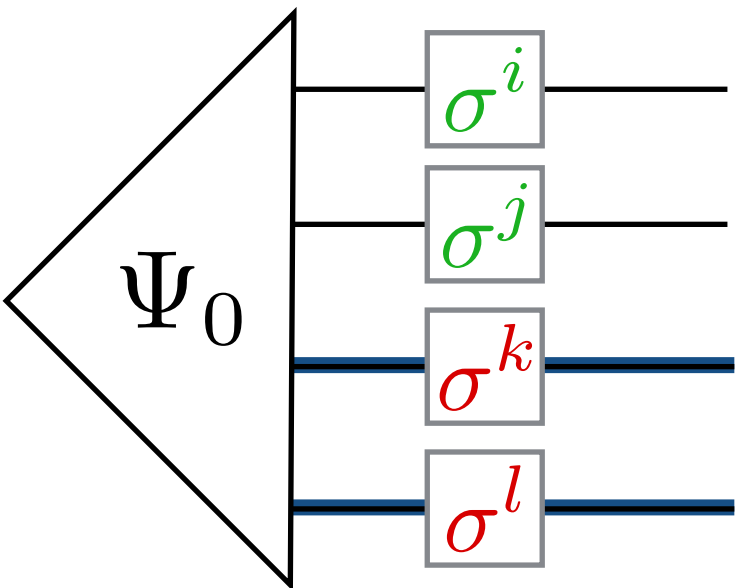
[[4,2,2]] Encoding

$$S_4 = \langle X X X X, Z Z Z Z \rangle$$

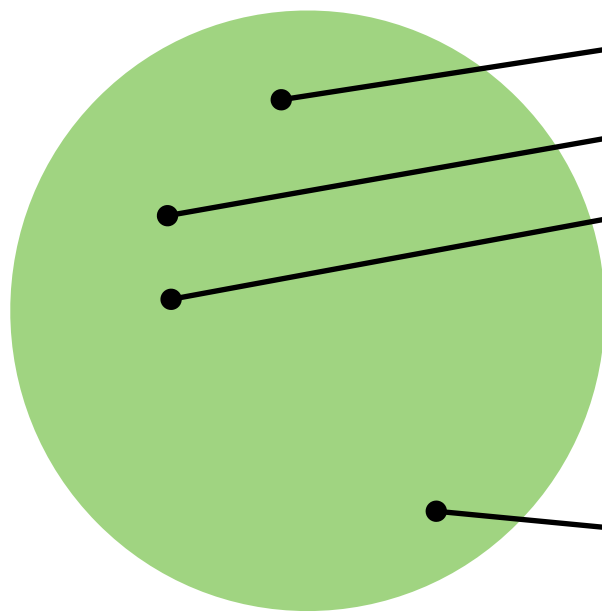
$$\bar{X}_1 = X X I I \quad \bar{Z}_1 = Z I Z I$$

$$\bar{X}_2 = I X I X \quad \bar{Z}_2 = I I Z Z$$

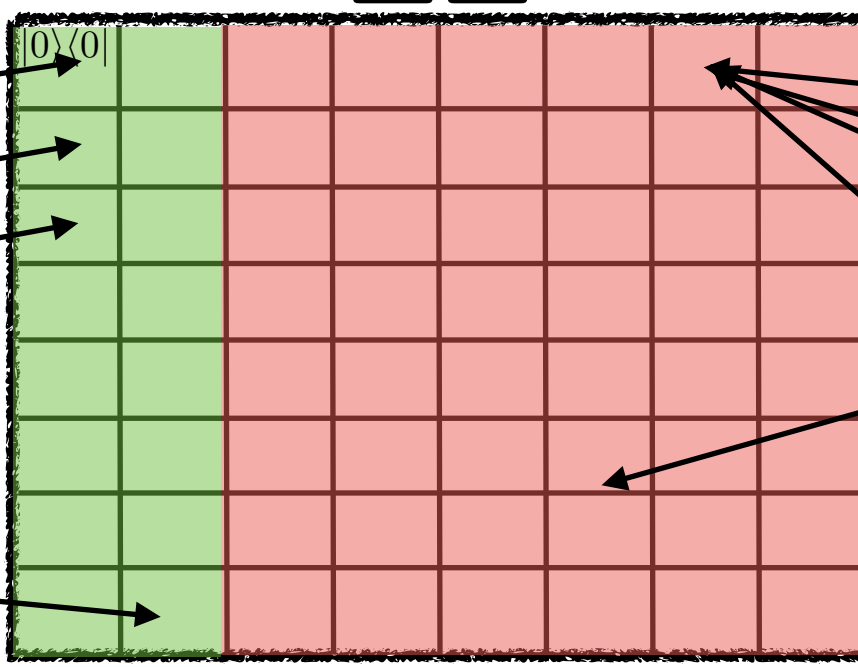
$$|0\rangle = |000000\rangle + |001111\rangle + |010101\rangle + |011010\rangle \\ + |100011\rangle + |101100\rangle + |110110\rangle + |111001\rangle$$



$$\sigma_1^i \sigma_2^j \otimes I^{\otimes 4}$$

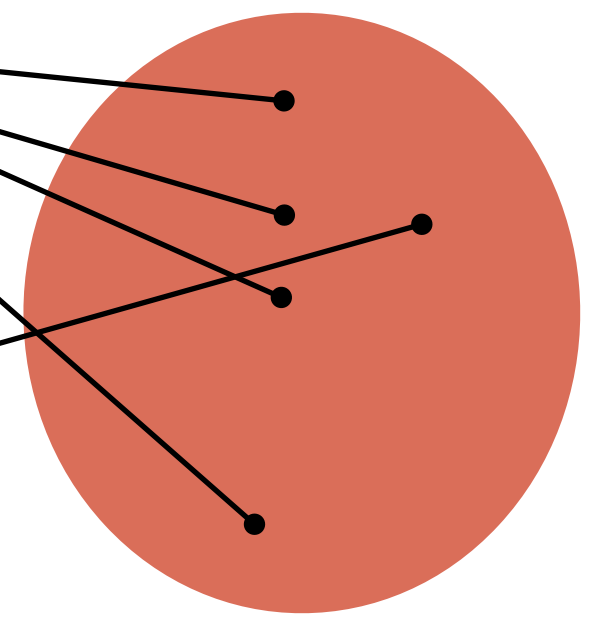


$I$



$H$

$$\sigma_1^i \sigma_2^j \otimes \sigma_z^k \otimes I^{\otimes 3}$$



# Asymmetric Code

$$\mathcal{S}_0 = \langle X I \bar{X} I, I X I \bar{X}, Z I \bar{Z} I, I Z I \bar{Z} \rangle$$

$$\mathcal{S}_1 = \langle I I X X X X, I I Z Z Z Z, X I X X I I, \\ Z I Z I Z I, I X I X I X, I Z I I Z Z \rangle.$$

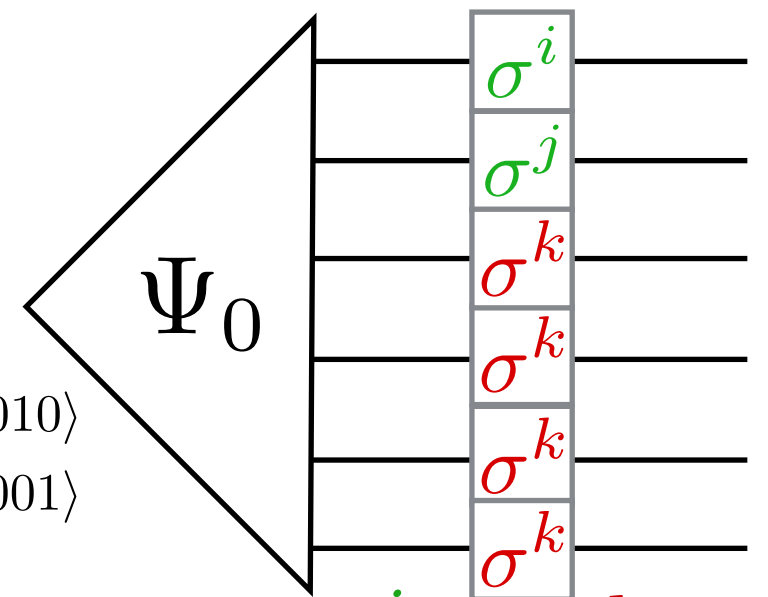
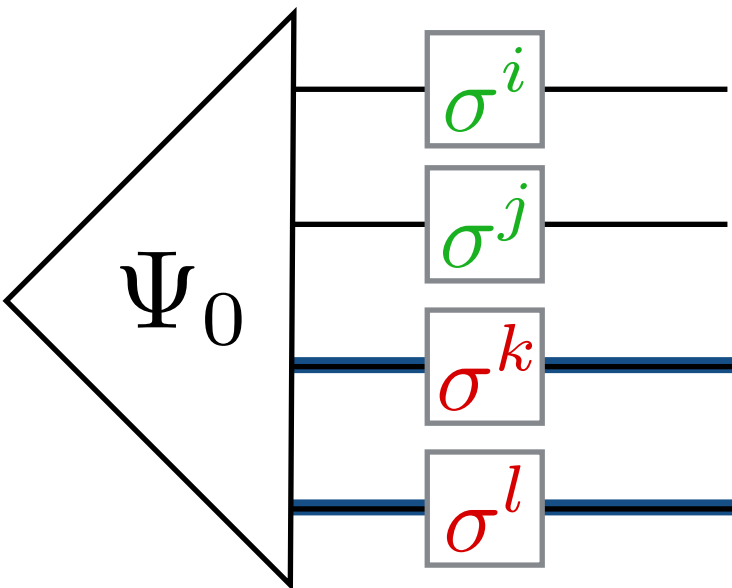
[[4,2,2]] Encoding

$$S_4 = \langle X X X X, Z Z Z Z \rangle$$

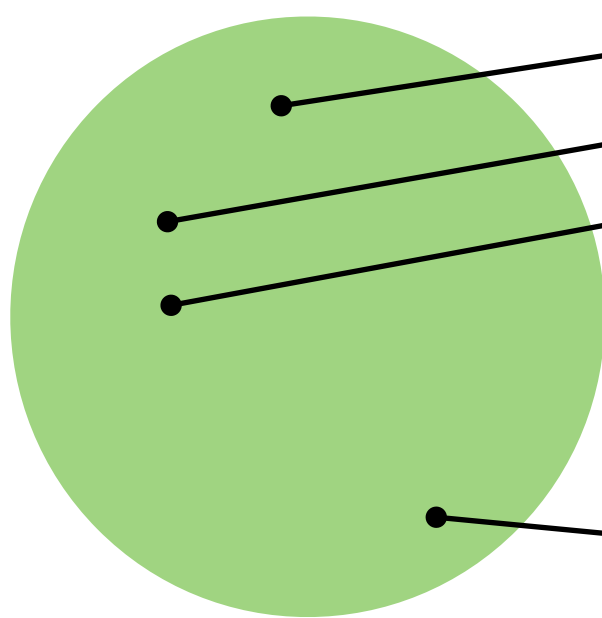
$$\bar{X}_1 = X X I I \quad \bar{Z}_1 = Z I Z I$$

$$\bar{X}_2 = I X I X \quad \bar{Z}_2 = I I Z Z$$

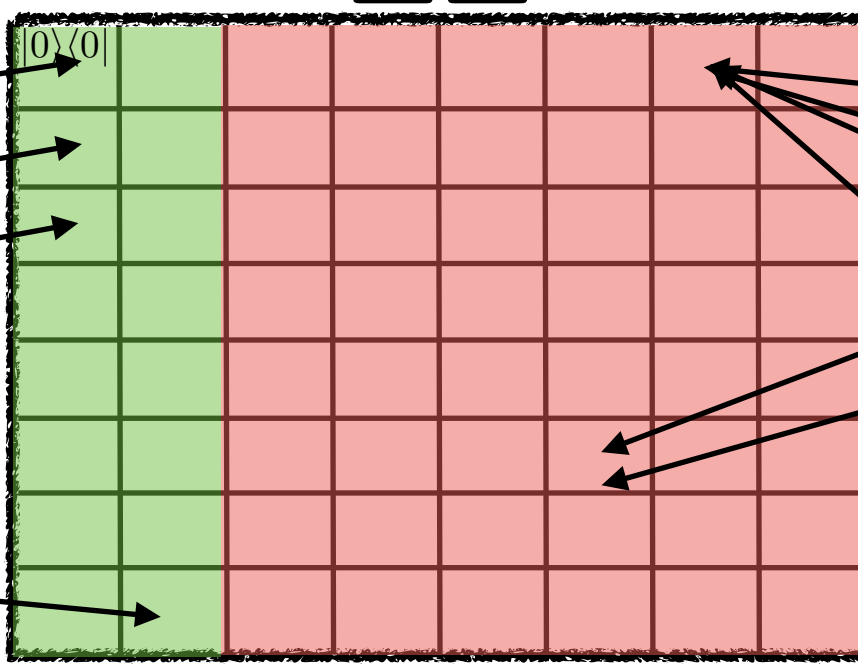
$$|0\rangle = |000000\rangle + |001111\rangle + |010101\rangle + |011010\rangle \\ + |100011\rangle + |101100\rangle + |110110\rangle + |111001\rangle$$



$$\sigma_1^i \sigma_2^j \otimes I^{\otimes 4}$$

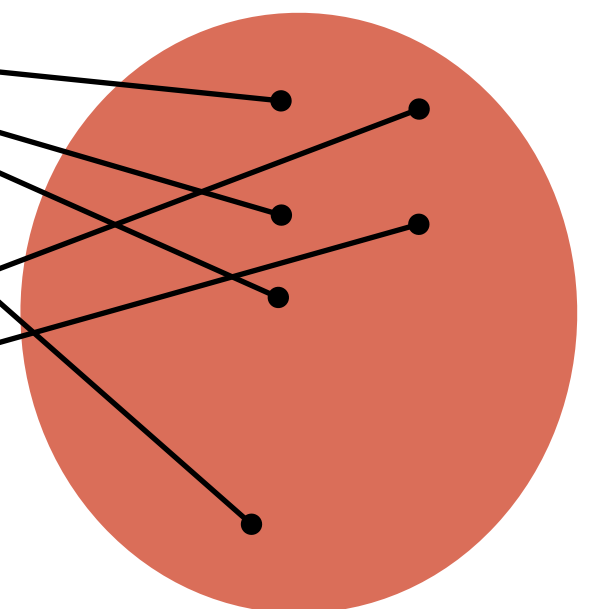


$I$



$H$

$$\sigma_1^i \sigma_2^j \otimes \sigma_z^k \otimes I^{\otimes 3}$$



# Asymmetric Code

$$\mathcal{S}_0 = \langle X I \bar{X} I, I X I \bar{X}, Z I \bar{Z} I, I Z I \bar{Z} \rangle$$

$$\mathcal{S}_1 = \langle I I X X X X, I I Z Z Z Z, X I X X I I, Z I Z I Z I, I X I X I X, I Z I I Z Z \rangle.$$

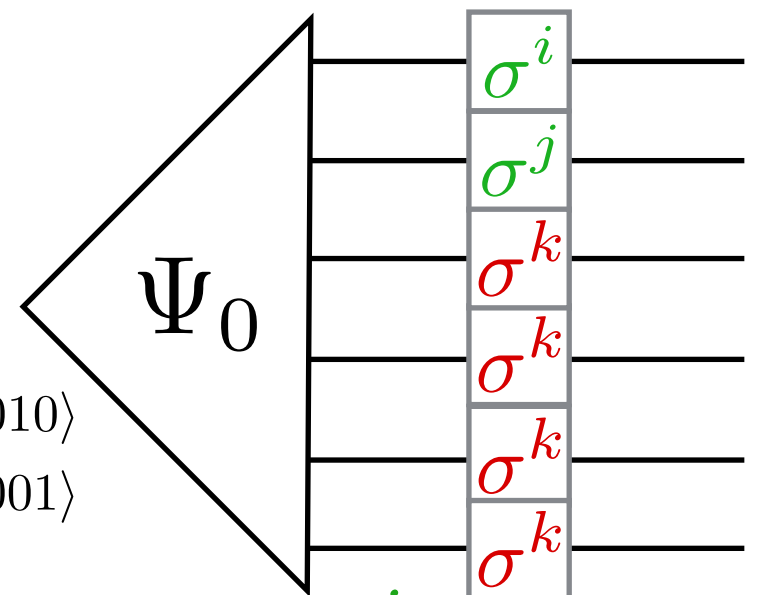
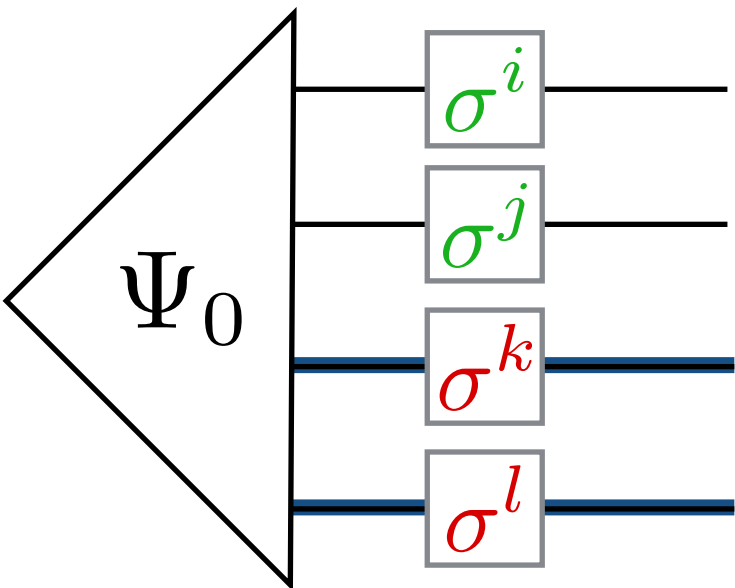
[[4,2,2]] Encoding

$$S_4 = \langle X X X X, Z Z Z Z \rangle$$

$$\bar{X}_1 = X X I I \quad \bar{Z}_1 = Z I Z I$$

$$\bar{X}_2 = I X I X \quad \bar{Z}_2 = I I Z Z$$

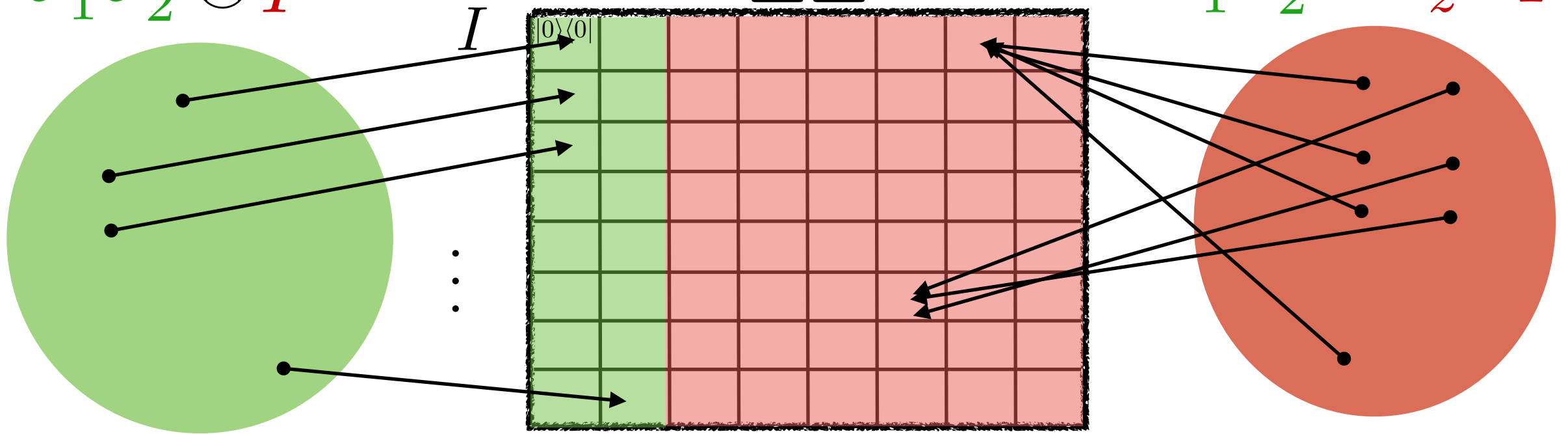
$$|0\rangle = |000000\rangle + |001111\rangle + |010101\rangle + |011010\rangle \\ + |100011\rangle + |101100\rangle + |110110\rangle + |111001\rangle$$



$$\sigma_1^i \sigma_2^j \otimes I^{\otimes 4}$$

$$\sigma_1^i \sigma_2^j \otimes \sigma_z^k \otimes I^{\otimes 3}$$

H



# Asymmetric Code

$$\mathcal{S}_0 = \langle X I \bar{X} I, I X I \bar{X}, Z I \bar{Z} I, I Z I \bar{Z} \rangle$$

$$\mathcal{S}_1 = \langle I I X X X X, I I Z Z Z Z, X I X X I I, Z I Z I Z I, I X I X I X, I Z I I Z Z \rangle.$$

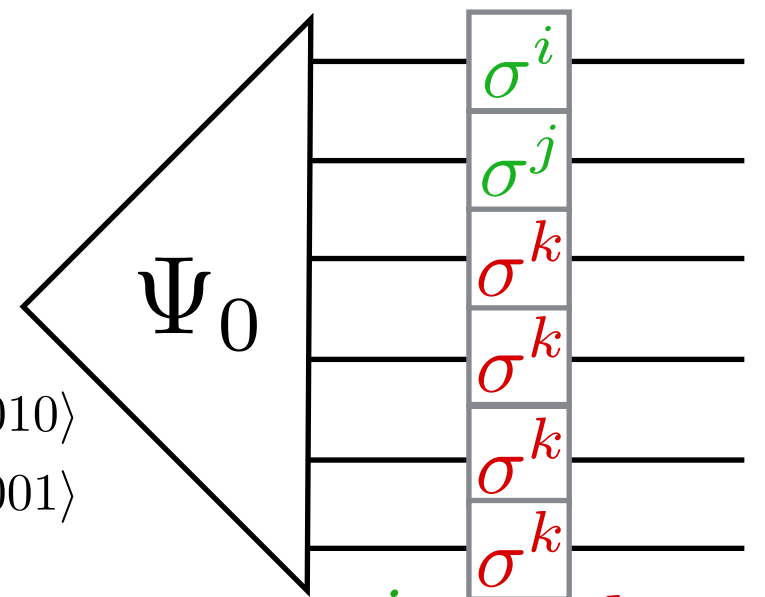
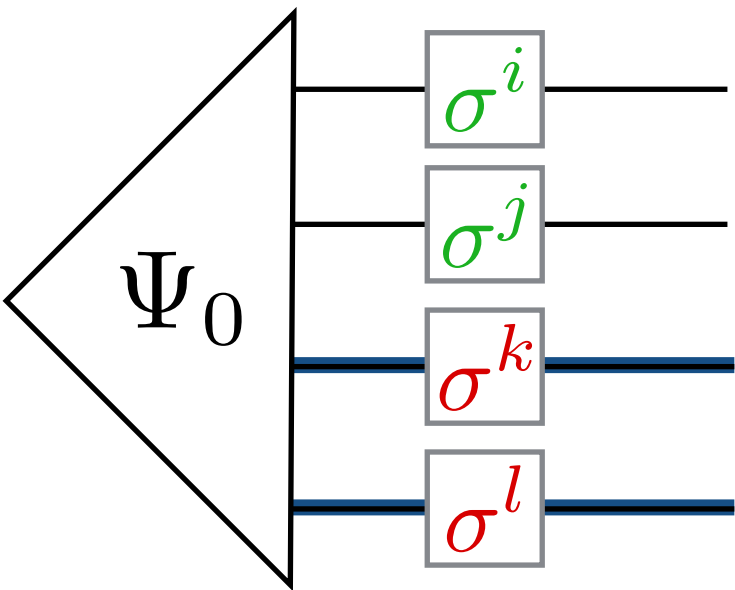
[[4,2,2]] Encoding

$$S_4 = \langle X X X X, Z Z Z Z \rangle$$

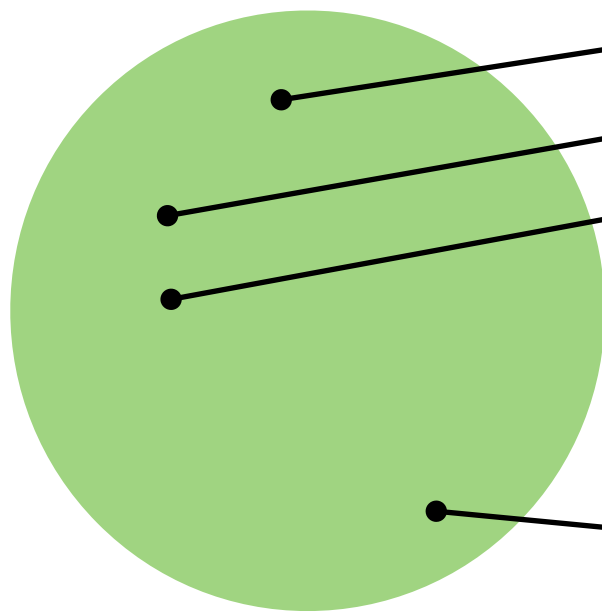
$$\bar{X}_1 = X X I I \quad \bar{Z}_1 = Z I Z I$$

$$\bar{X}_2 = I X I X \quad \bar{Z}_2 = I I Z Z$$

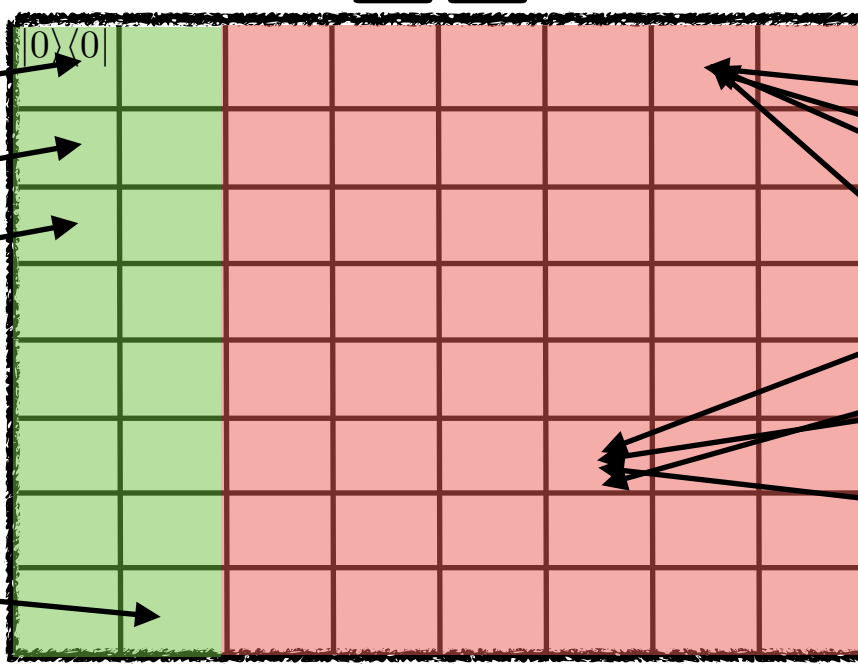
$$|0\rangle = |000000\rangle + |001111\rangle + |010101\rangle + |011010\rangle \\ + |100011\rangle + |101100\rangle + |110110\rangle + |111001\rangle$$



$$\sigma_1^i \sigma_2^j \otimes I^{\otimes 4}$$

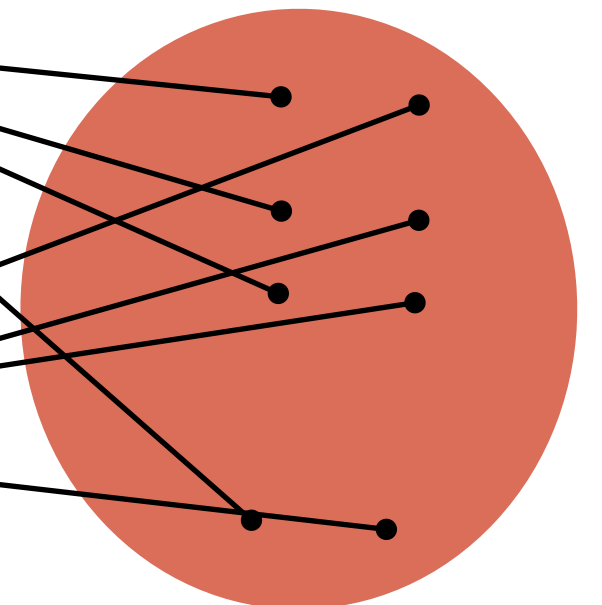


$I$

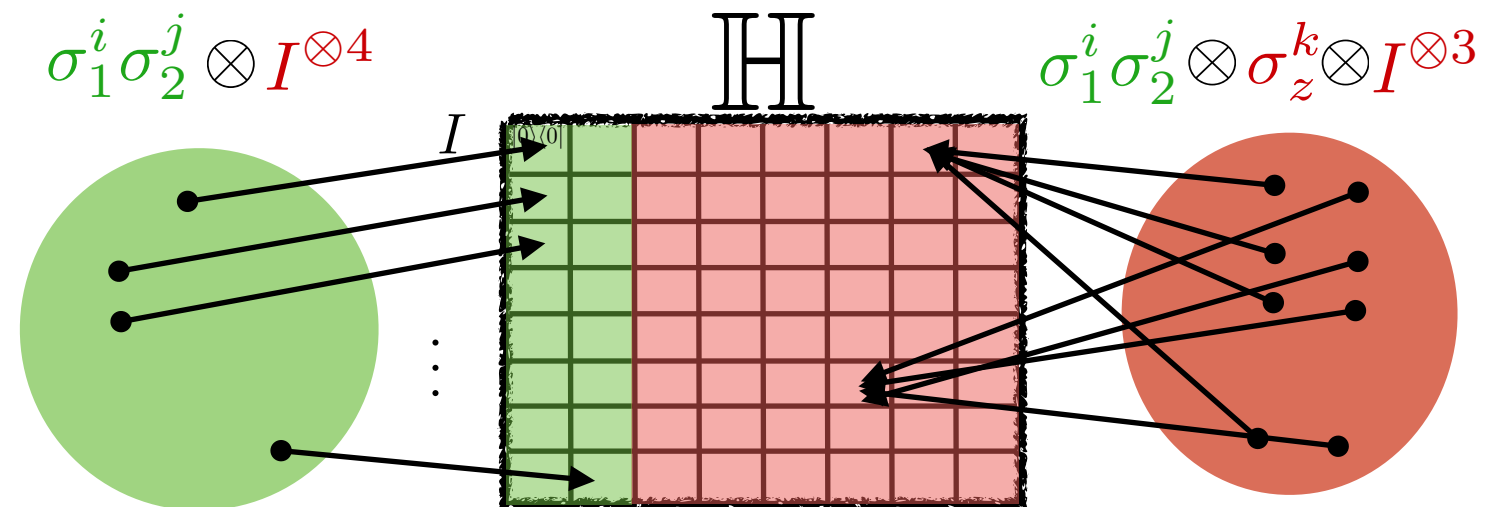


$H$

$$\sigma_1^i \sigma_2^j \otimes \sigma_z^k \otimes I^{\otimes 3}$$



# Selective Reconstruction



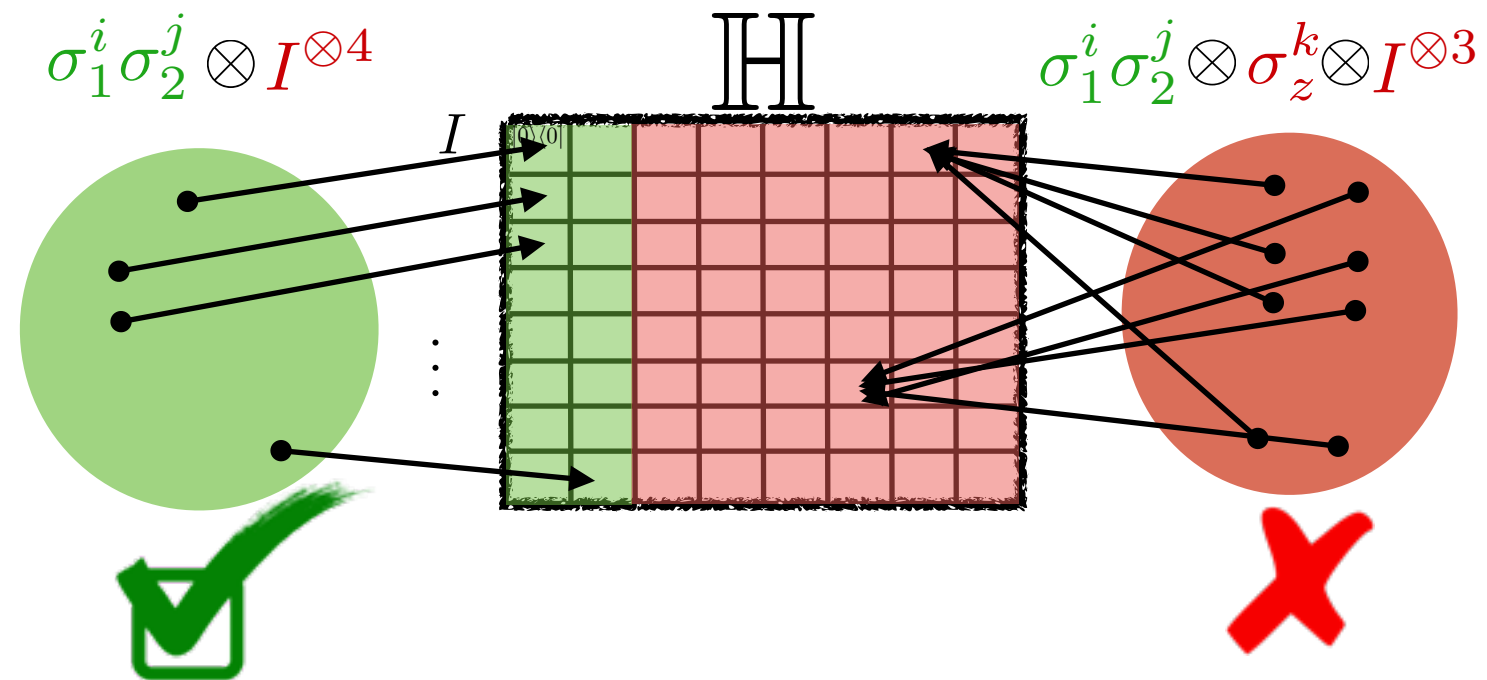


# Selective Reconstruction

$$\mathcal{S}_1 = \langle IIXXXX, IIZZZZ, XIXXII, \\ ZIZIZI, IXIXIX, IZIIZZ \rangle.$$

$i$	$E_i$	$e_i$	$i$	$E_i$	$e_i$
0	11	000000	8	XY	000111
1	X1	000100	9	XZ	000110
2	Y1	001100	10	YX	001101
3	Z1	001000	11	YY	001111
4	1X	000001	12	YZ	001110
5	1Y	000011	13	ZX	001001
6	1Z	000010	14	ZY	001011
7	XX	000101	15	ZZ	001010

**Index**      **Operators**      **Signature**

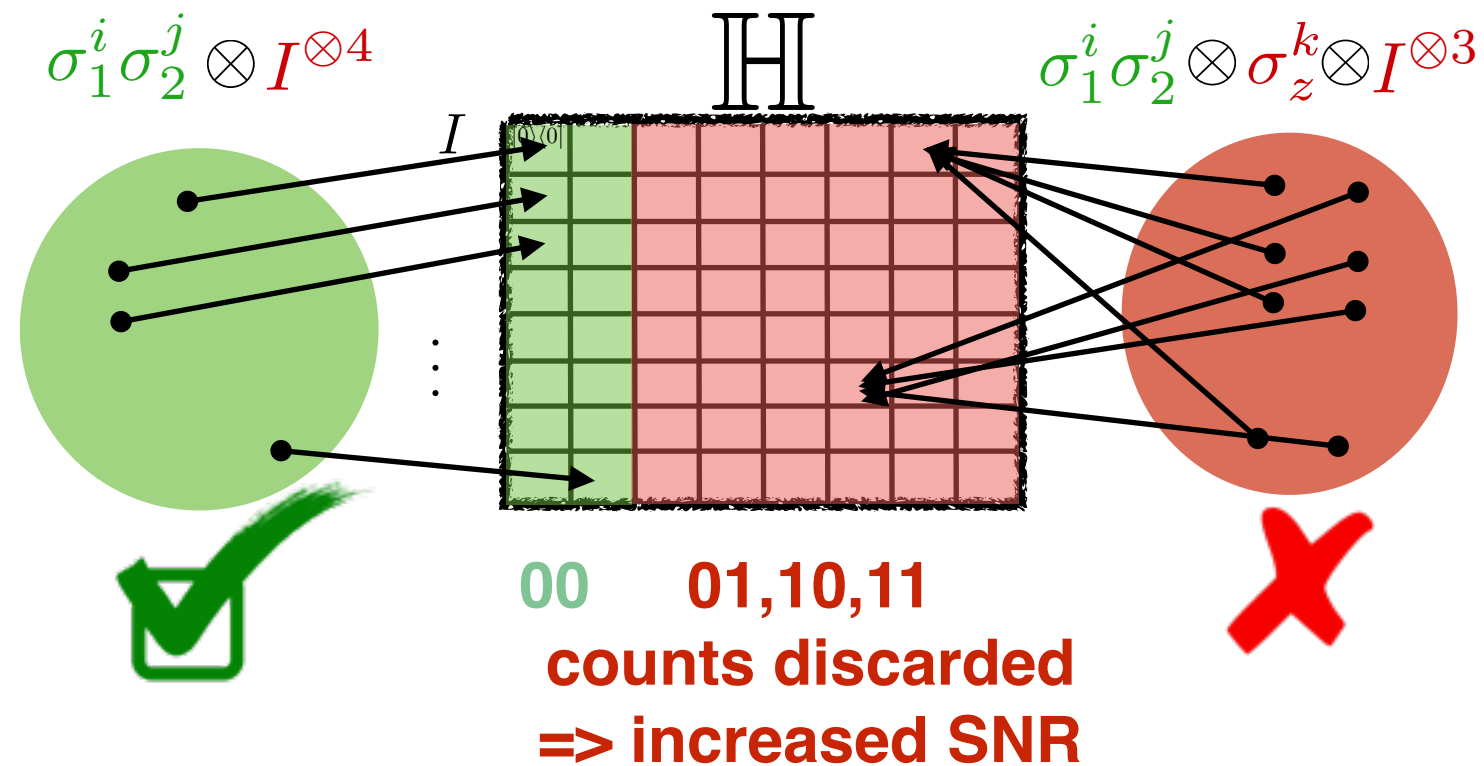


# Selective Reconstruction

$$\mathcal{S}_1 = \langle IIXXXX, IIZZZZ, XIXXII, \\ ZIZIZI, IXIXIX, IZIIZZ \rangle.$$

$i$	$E_i$	$e_i$	$i$	$E_i$	$e_i$
0	11	000000	8	XY	000111
1	X1	000100	9	XZ	000110
2	Y1	001100	10	YX	001101
3	Z1	001000	11	YY	001111
4	1X	000001	12	YZ	001110
5	1Y	000011	13	ZX	001001
6	1Z	000010	14	ZY	001011
7	XX	000101	15	ZZ	001010

**Index**      **Operators**      **Signature**

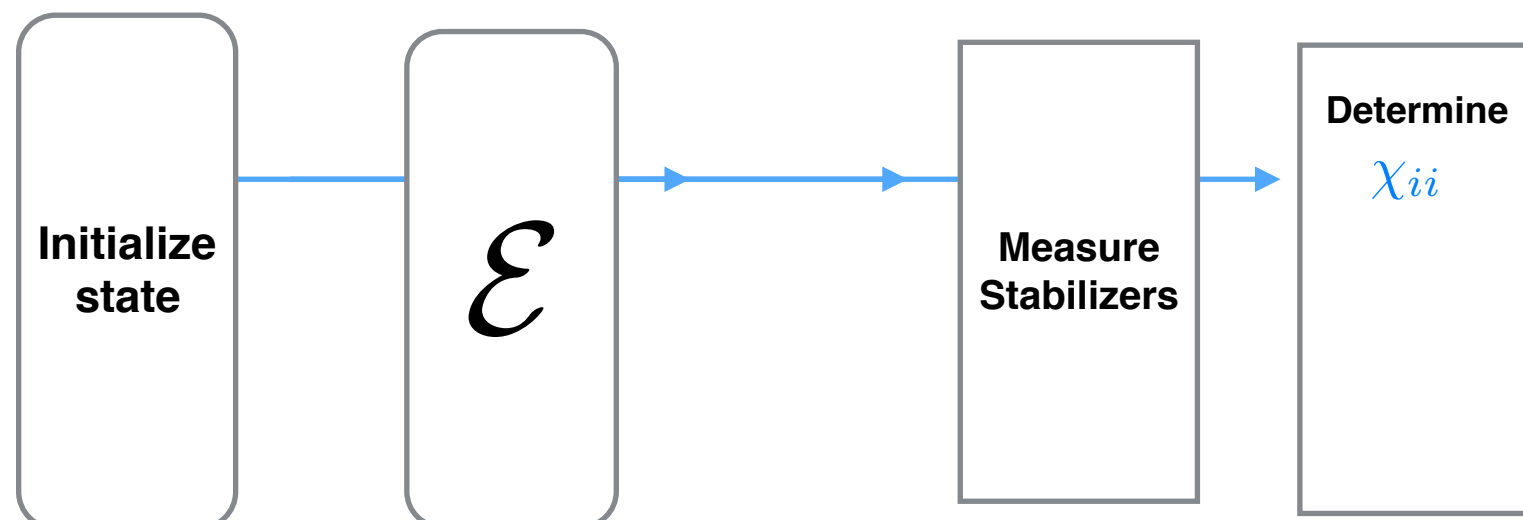
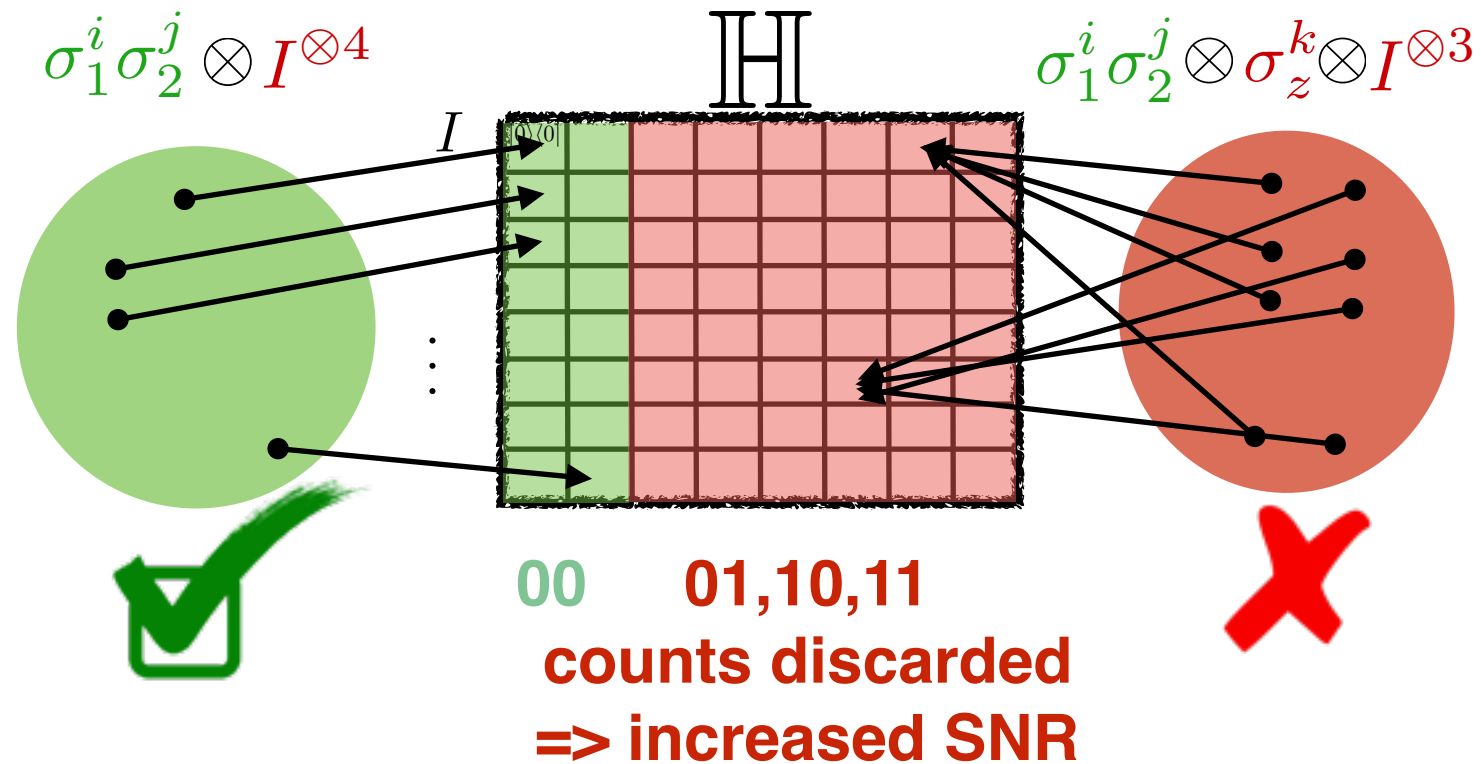


# Selective Reconstruction

$$\mathcal{S}_1 = \langle IIXXXX, IIZZZZ, XIXXII, \\ ZIZIZI, IXIXIX, IZIIZZ \rangle.$$

$i$	$E_i$	$e_i$	$i$	$E_i$	$e_i$
0	11	000000	8	XY	000111
1	X1	000100	9	XZ	000110
2	Y1	001100	10	YX	001101
3	Z1	001000	11	YY	001111
4	1X	000001	12	YZ	001110
5	1Y	000011	13	ZX	001001
6	1Z	000010	14	ZY	001011
7	XX	000101	15	ZZ	001010

**Index**      **Operators**      **Signature**



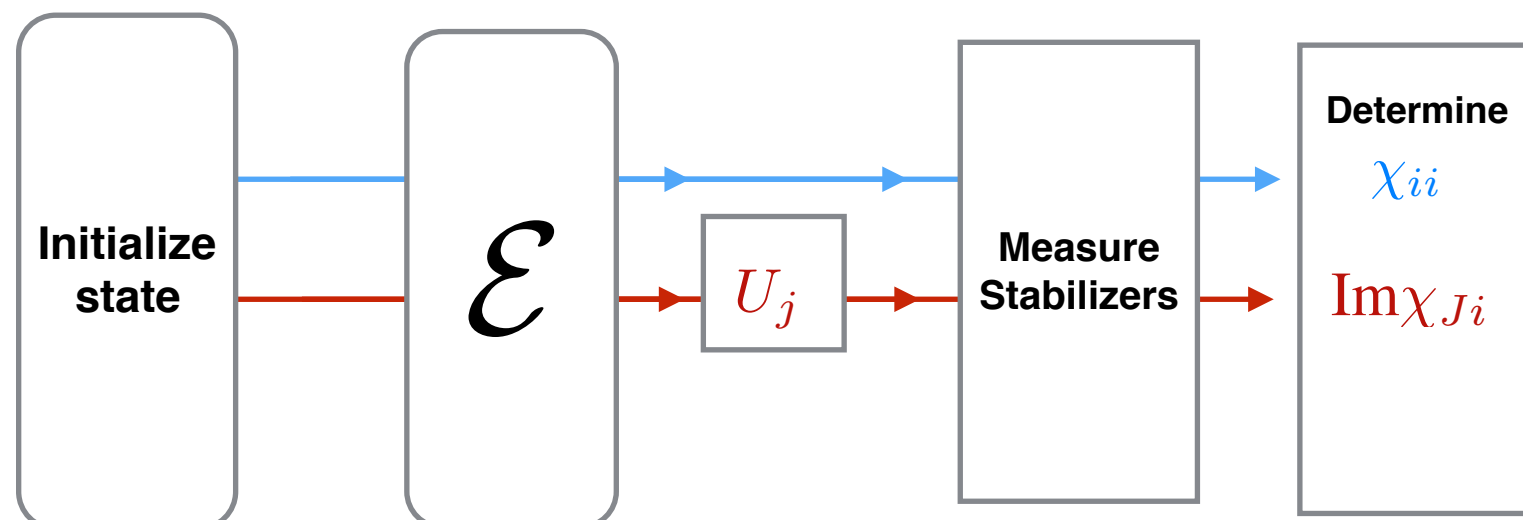
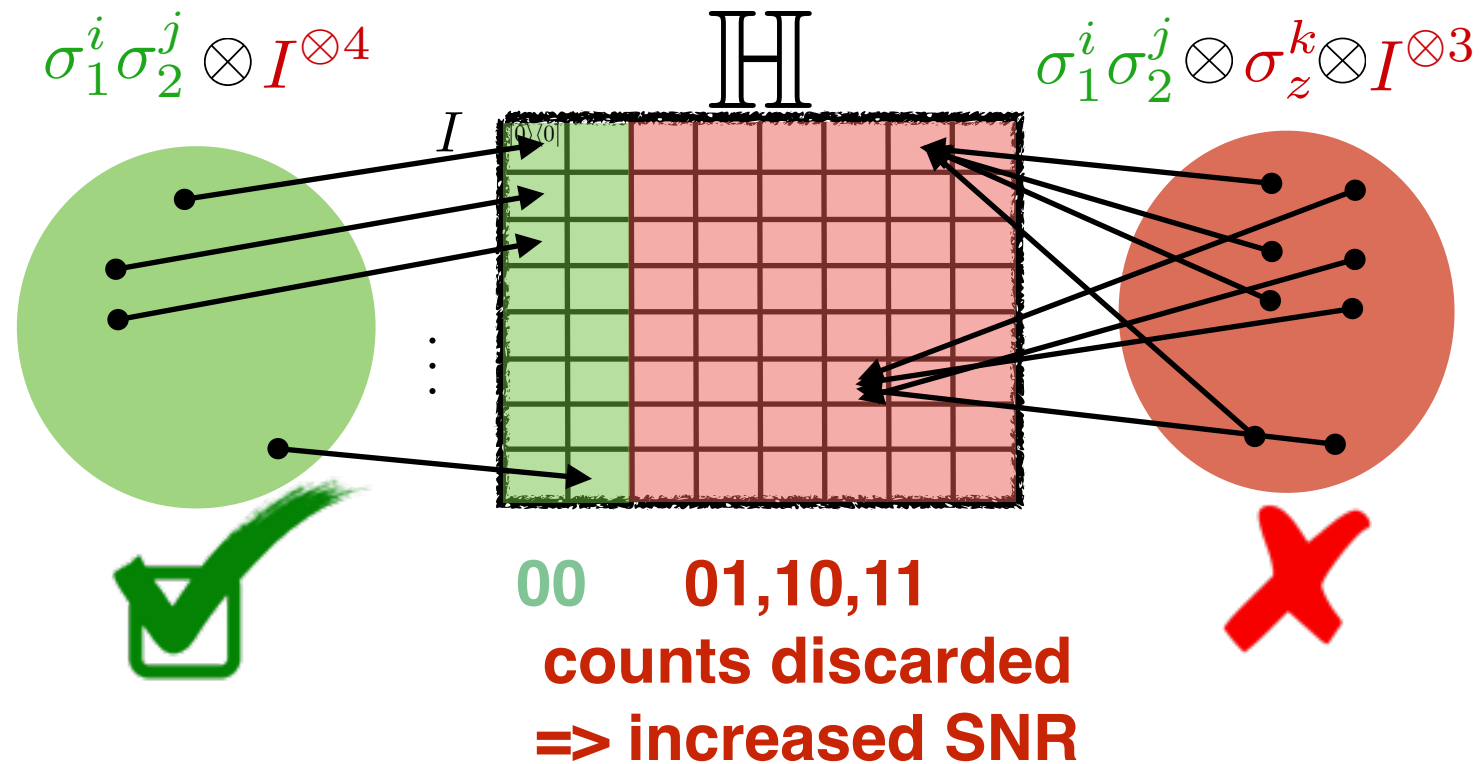
$$p_i = \chi_{ii}$$

# Selective Reconstruction

$$\mathcal{S}_1 = \langle IIXXXX, IIZZZZ, XIXXII, \\ ZIZIZI, IXIXIX, IZIIZZ \rangle.$$

$i$	$E_i$	$e_i$	$i$	$E_i$	$e_i$
0	11	000000	8	XY	000111
1	X1	000100	9	XZ	000110
2	Y1	001100	10	YX	001101
3	Z1	001000	11	YY	001111
4	1X	000001	12	YZ	001110
5	1Y	000011	13	ZX	001001
6	1Z	000010	14	ZY	001011
7	XX	000101	15	ZZ	001010

**Index**      **Operators**      **Signature**



$$p_i = \chi_{ii}$$

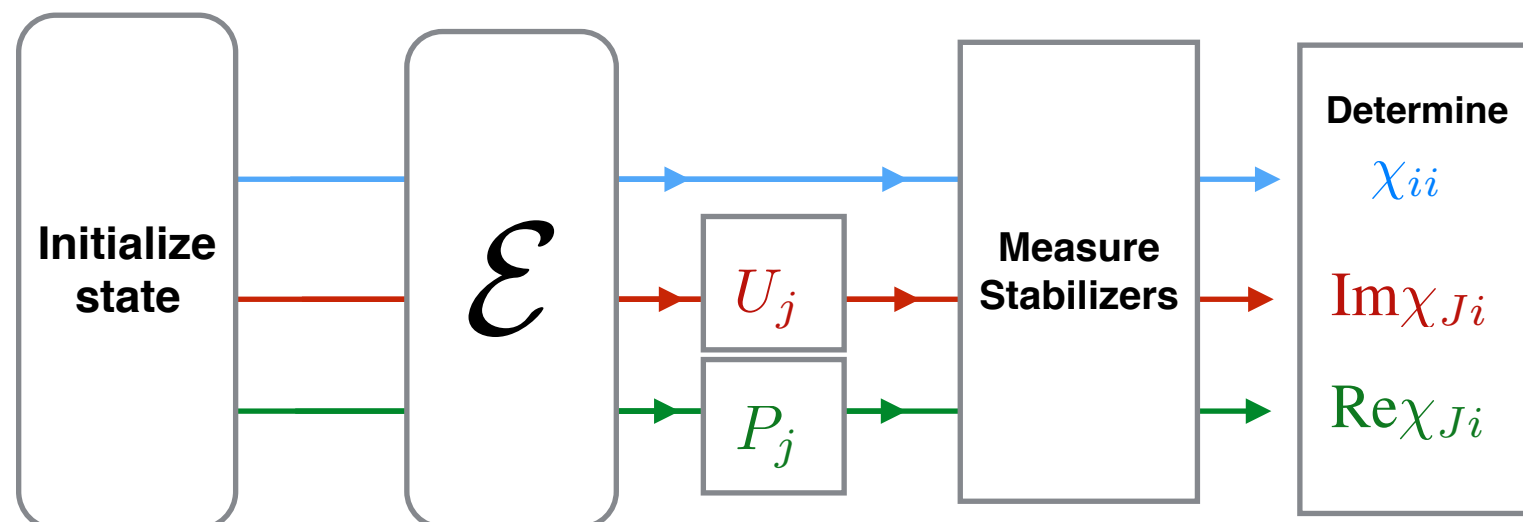
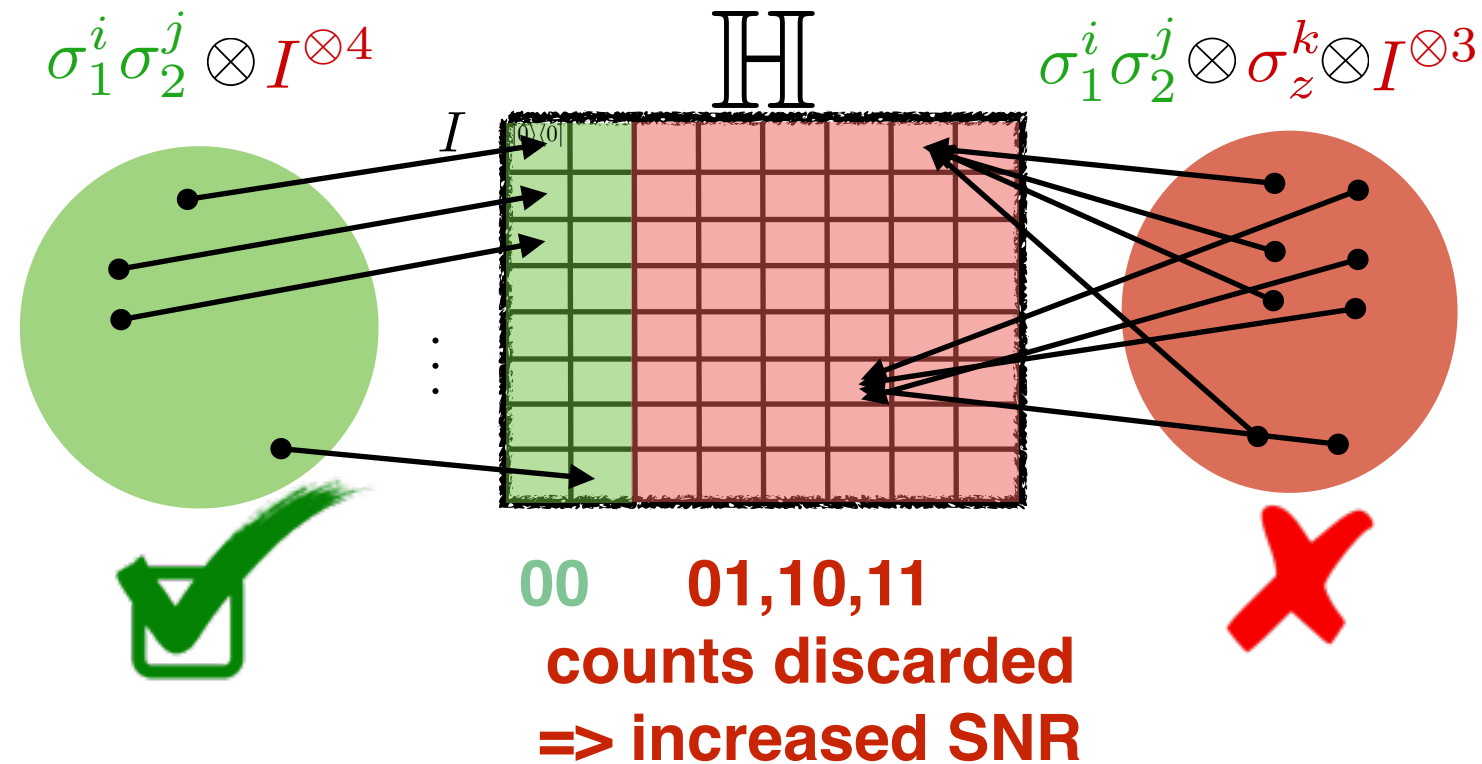
$$p_i(U_j) = \frac{\chi_{ii} + \chi_{JJ}}{2} - \text{Im}(\phi_J \chi_{Ji})$$

# Selective Reconstruction

$$\mathcal{S}_1 = \langle IIXXXX, IIZZZZ, XIXXII, \\ ZIZIZI, IXIXIX, IZIIZZ \rangle.$$

$i$	$E_i$	$e_i$	$i$	$E_i$	$e_i$
0	11	000000	8	XY	000111
1	X1	000100	9	XZ	000110
2	Y1	001100	10	YX	001101
3	Z1	001000	11	YY	001111
4	1X	000001	12	YZ	001110
5	1Y	000011	13	ZX	001001
6	1Z	000010	14	ZY	001011
7	XX	000101	15	ZZ	001010

Index
Operators
Signature



$$p_i = \chi_{ii}$$

$$p_i(U_j) = \frac{\chi_{ii} + \chi_{JJ}}{2} - \text{Im}(\phi_J \chi_{Ji})$$

$$p_i(P_{j\pm}) = \frac{\chi_{ii} + \chi_{JJ}}{2} \pm \text{Re}(\phi_J \chi_{Ji})$$

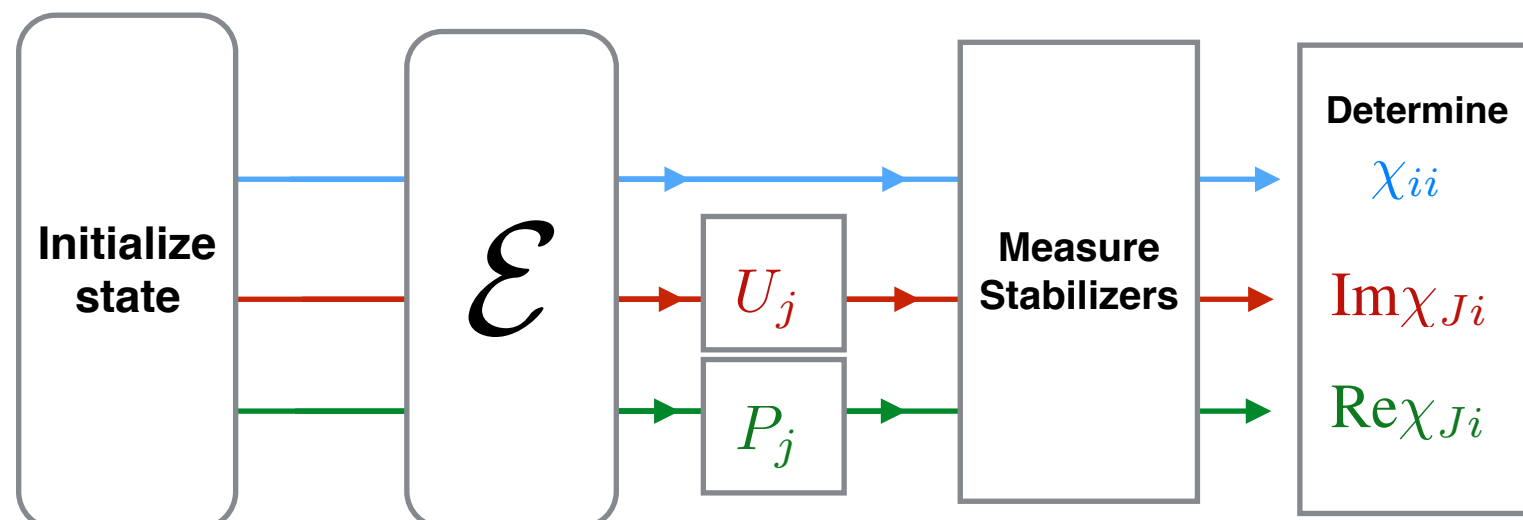
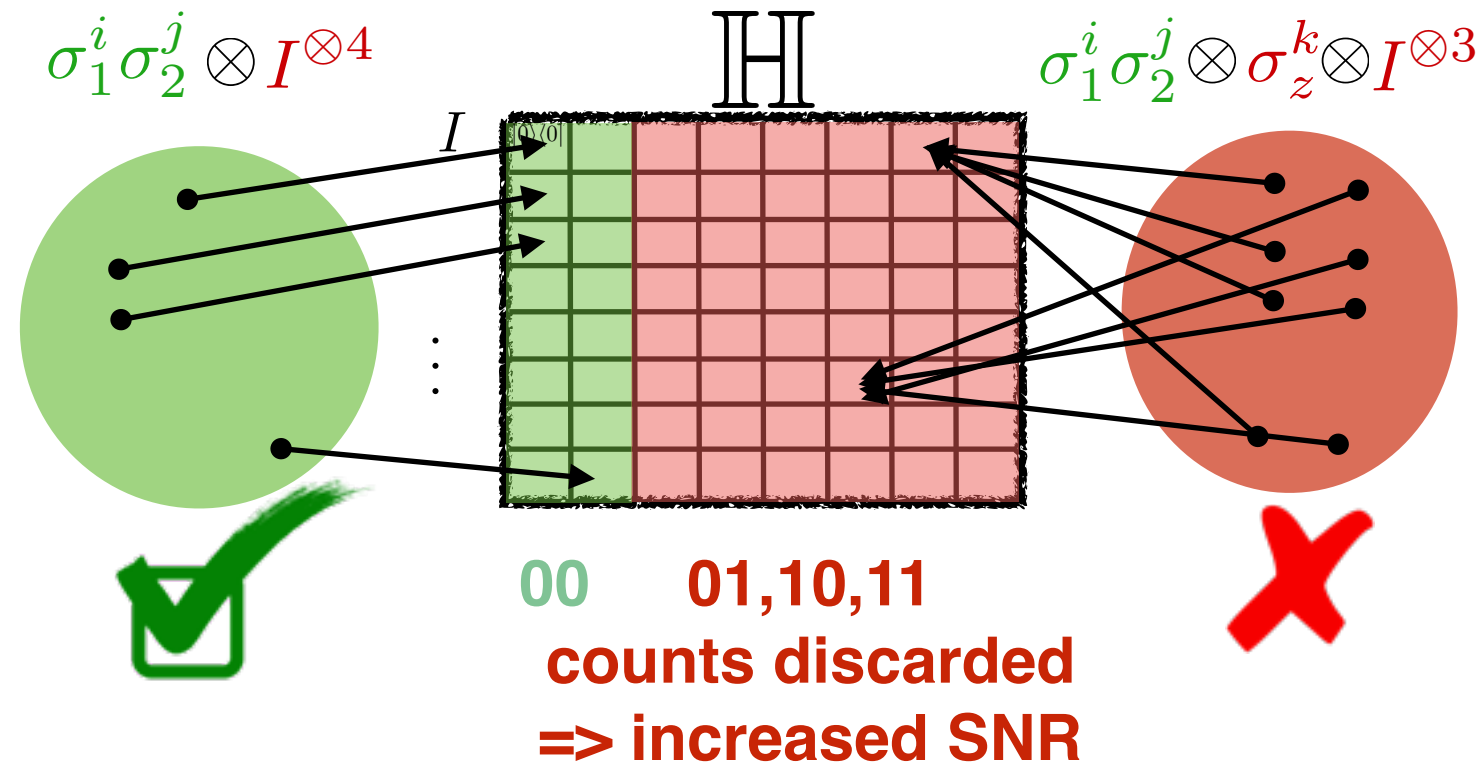


# Selective Reconstruction

$$\mathcal{S}_1 = \langle IIXXXX, IIZZZZ, XIXXII, \\ ZIZIZI, IXIXIX, IZIIZZ \rangle.$$

$i$	$E_i$	$e_i$	$i$	$E_i$	$e_i$
0	11	000000	8	XY	000111
1	X1	000100	9	XZ	000110
2	Y1	001100	10	YX	001101
3	Z1	001000	11	YY	001111
4	1X	000001	12	YZ	001110
5	1Y	000011	13	ZX	001001
6	1Z	000010	14	ZY	001011
7	XX	000101	15	ZZ	001010

Index
Operators
Signature



$$p_i = \chi_{ii}$$

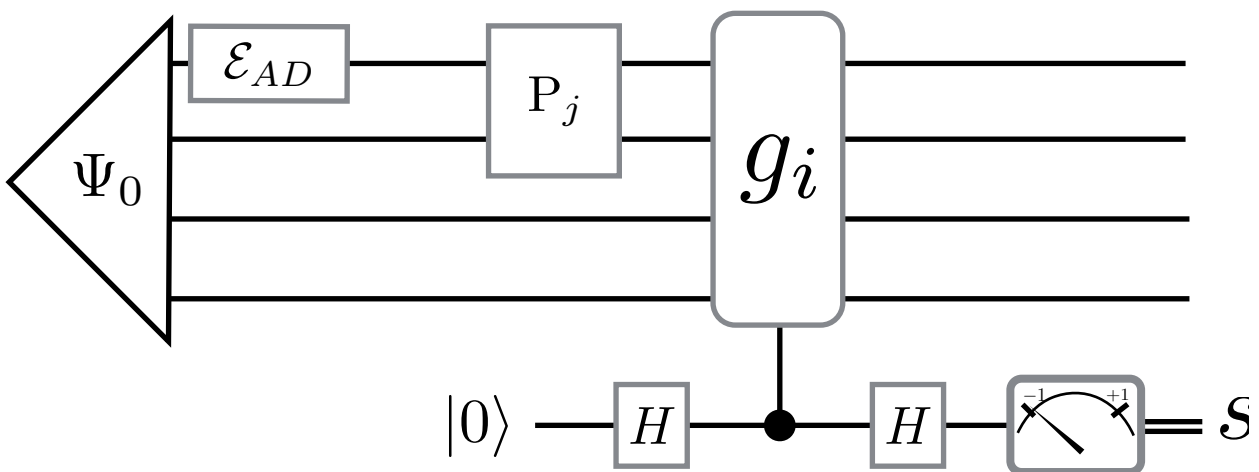
$$p_i(U_j) = \frac{\chi_{ii} + \chi_{JJ}}{2} - \text{Im}(\phi_J \chi_{Ji})$$

$$p_i(P_{j\pm}) = \frac{\chi_{ii} + \chi_{JJ}}{2} \pm \text{Re}(\phi_J \chi_{Ji})$$

$$\mathcal{E}(\rho) = \sum_{m,n} \chi_{mn} F_i \rho F_i^\dagger$$

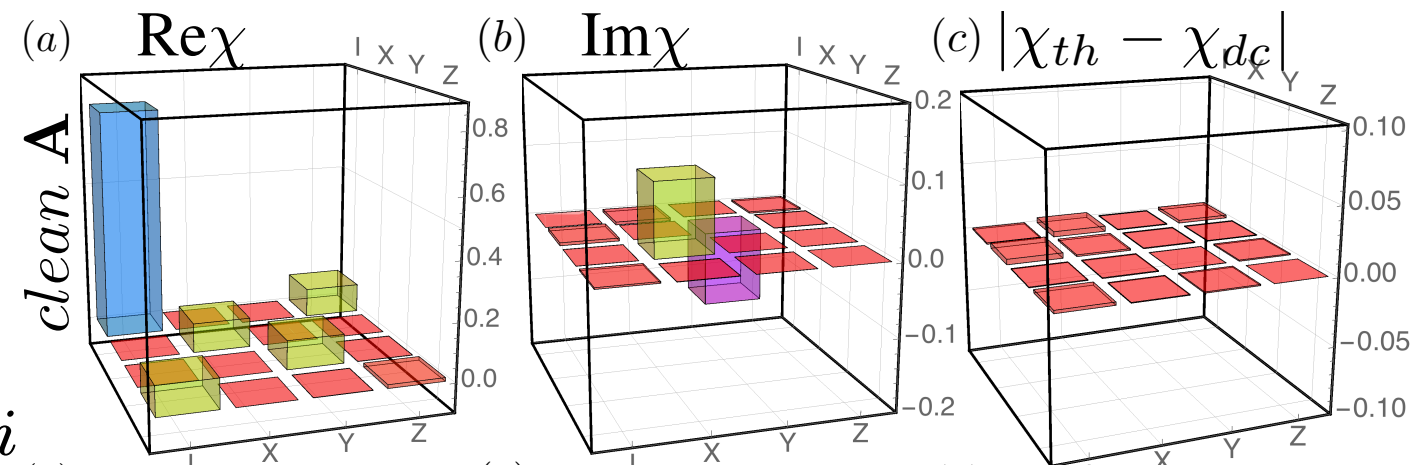
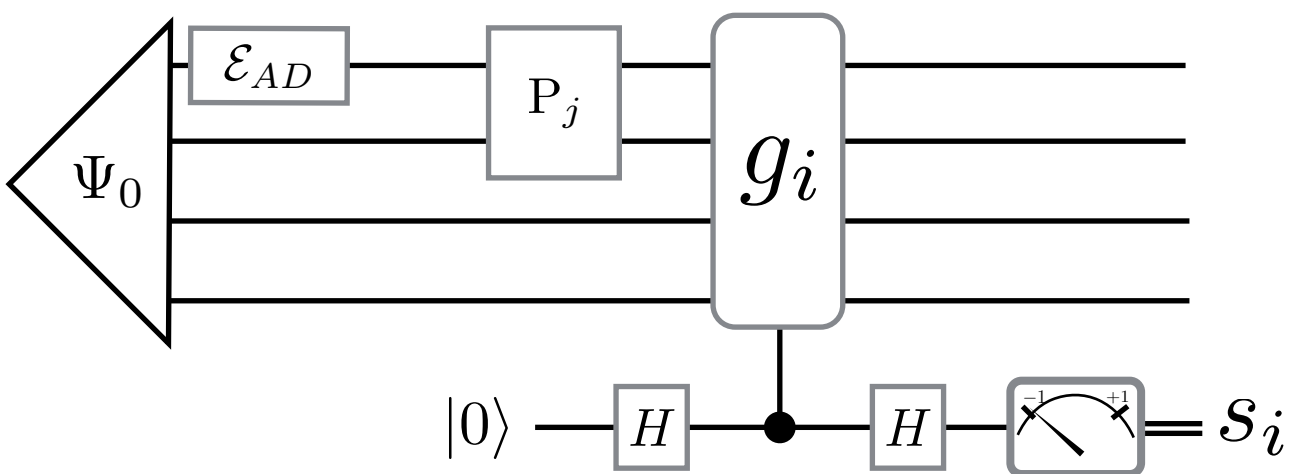
# Ex: Single Qubit Process

# Ex: Single Qubit Process



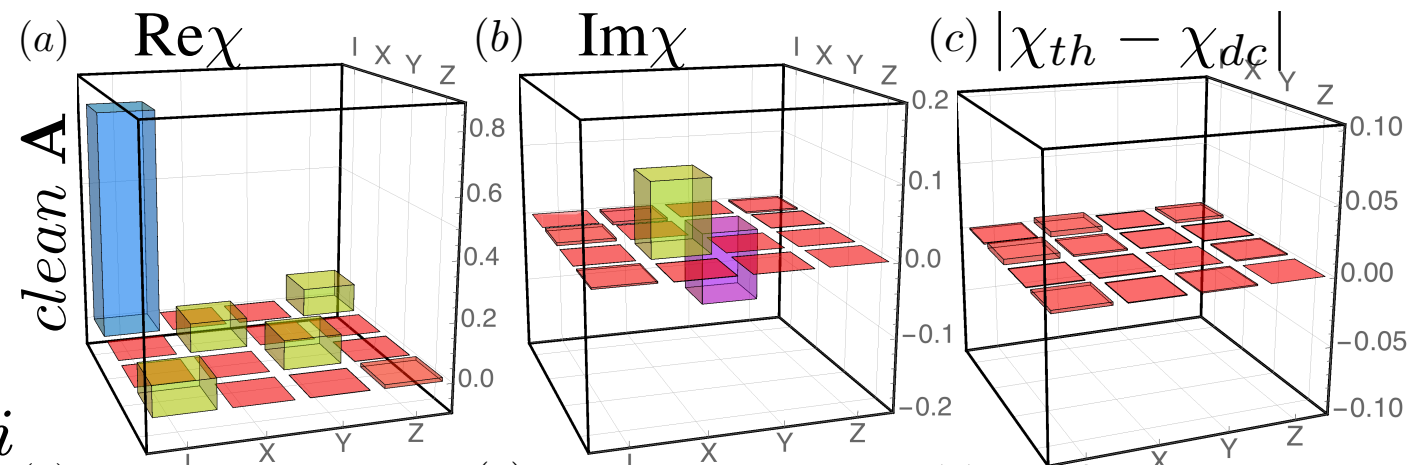
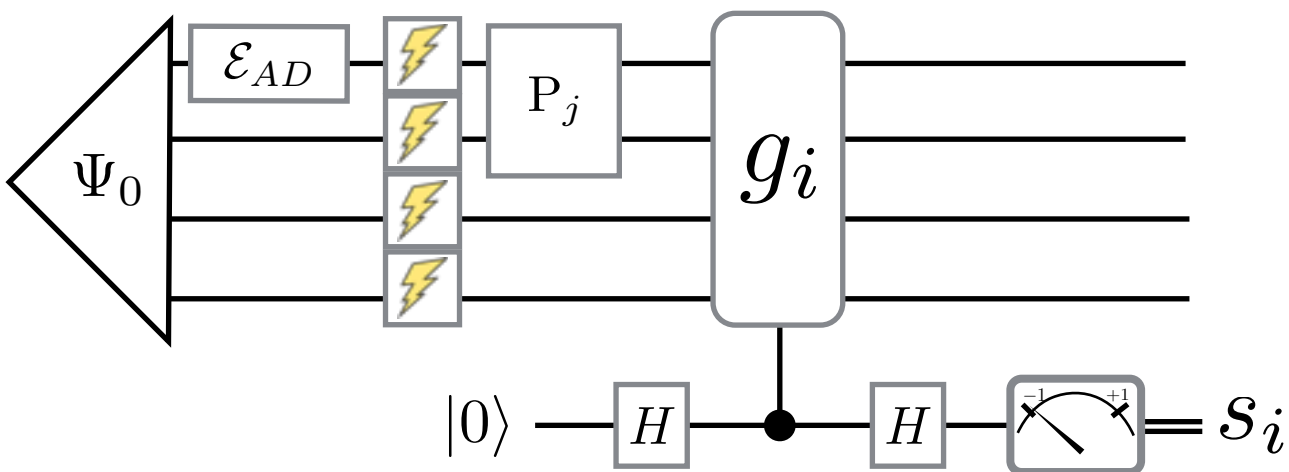


# Ex: Single Qubit Process



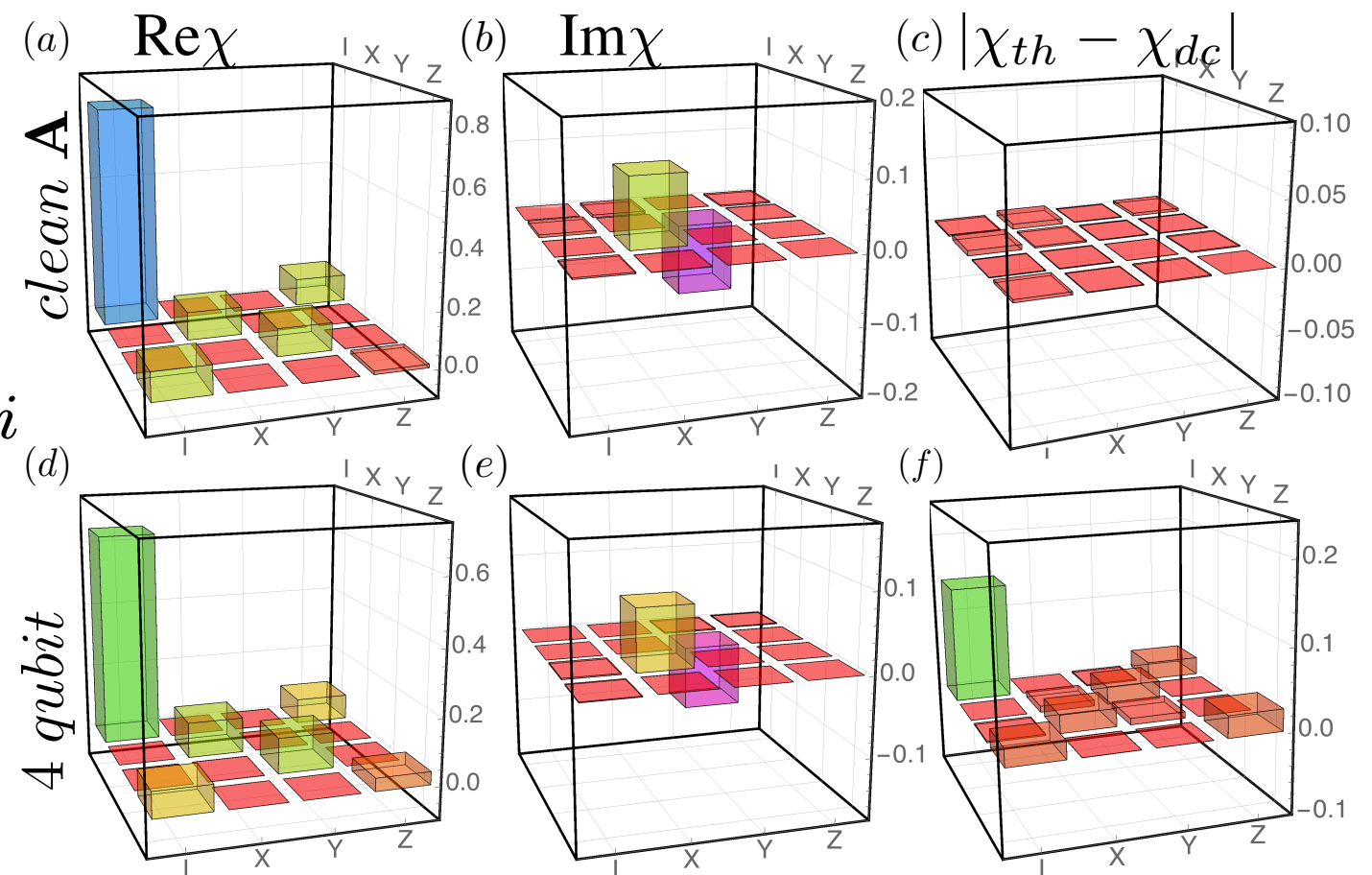
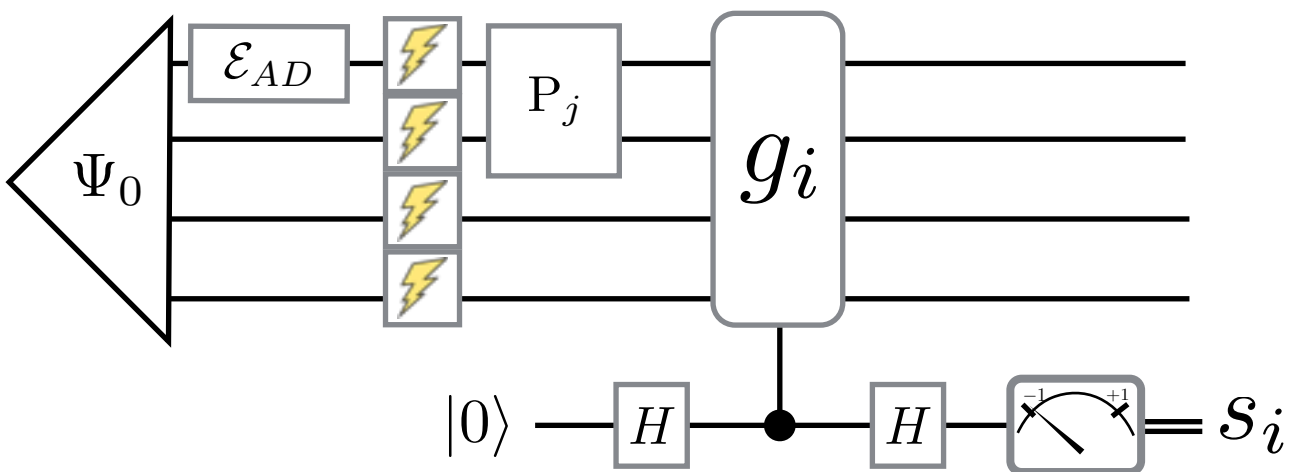
# Ex: Single Qubit Process

Depolarizing Noise  $p_{DP} = 0.1$



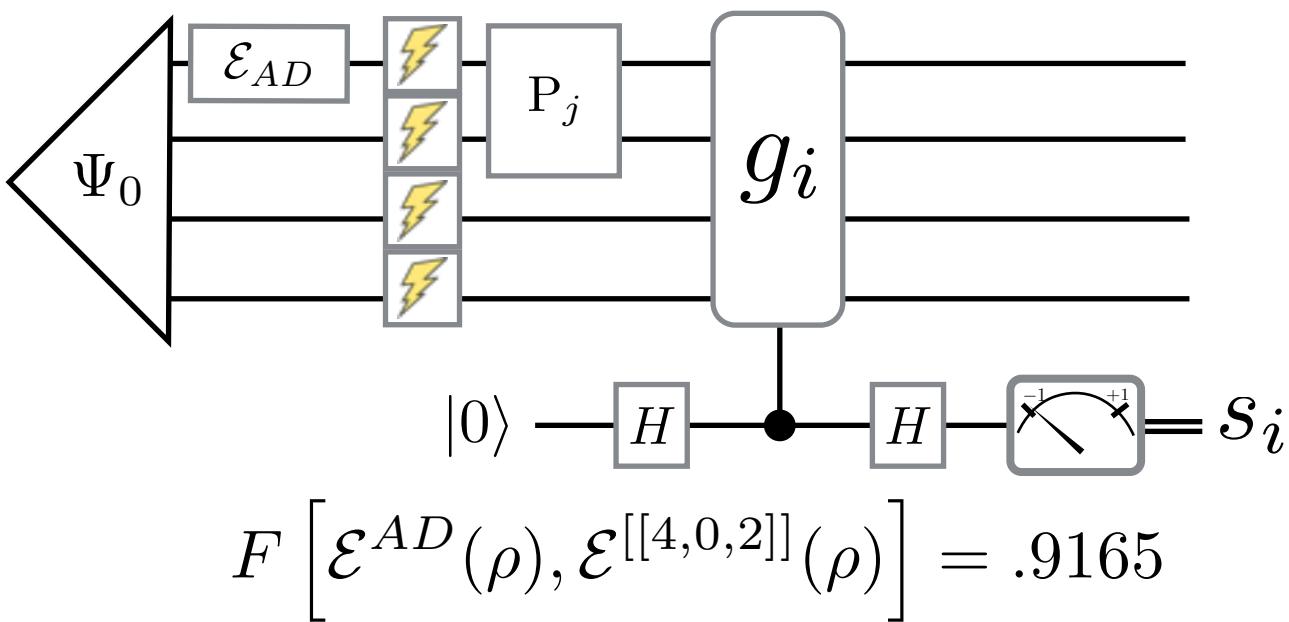
# Ex: Single Qubit Process

Depolarizing Noise  $p_{DP} = 0.1$

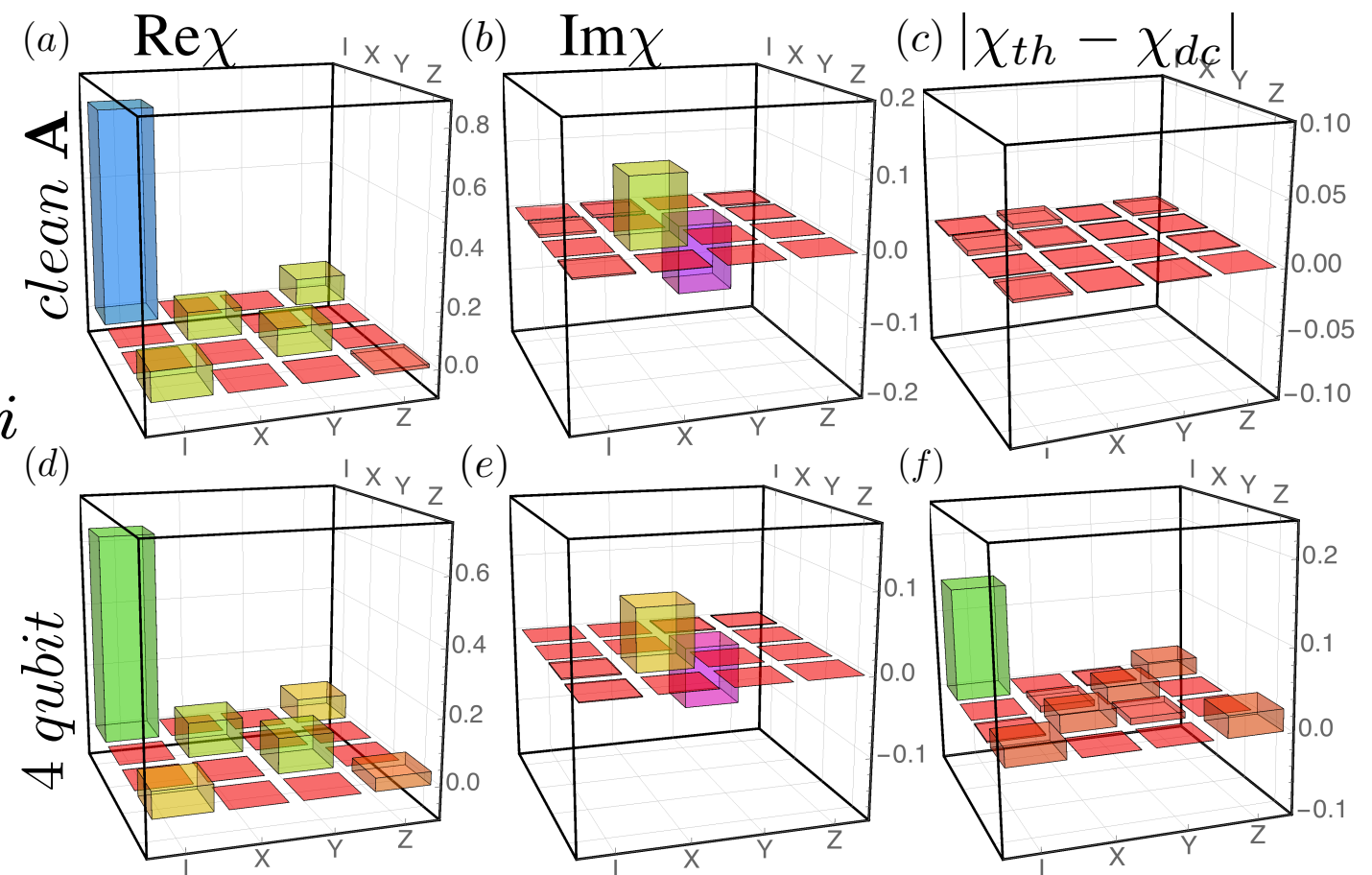


# Ex: Single Qubit Process

Depolarizing Noise  $p_{DP} = 0.1$

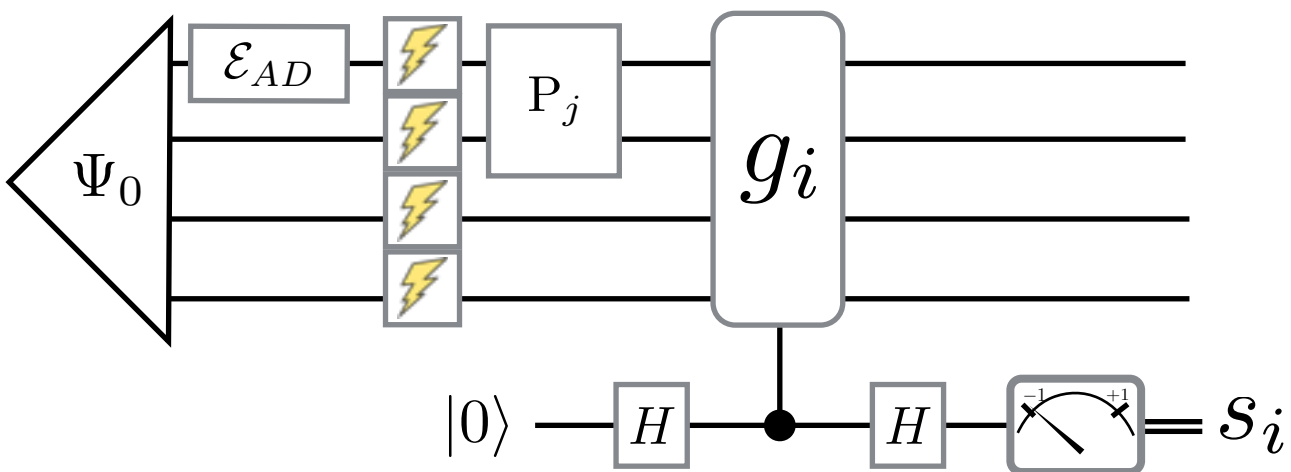


$$F \left[ \mathcal{E}^{AD}(\rho), \mathcal{E}^{[[4,0,2]]}(\rho) \right] = .9165$$

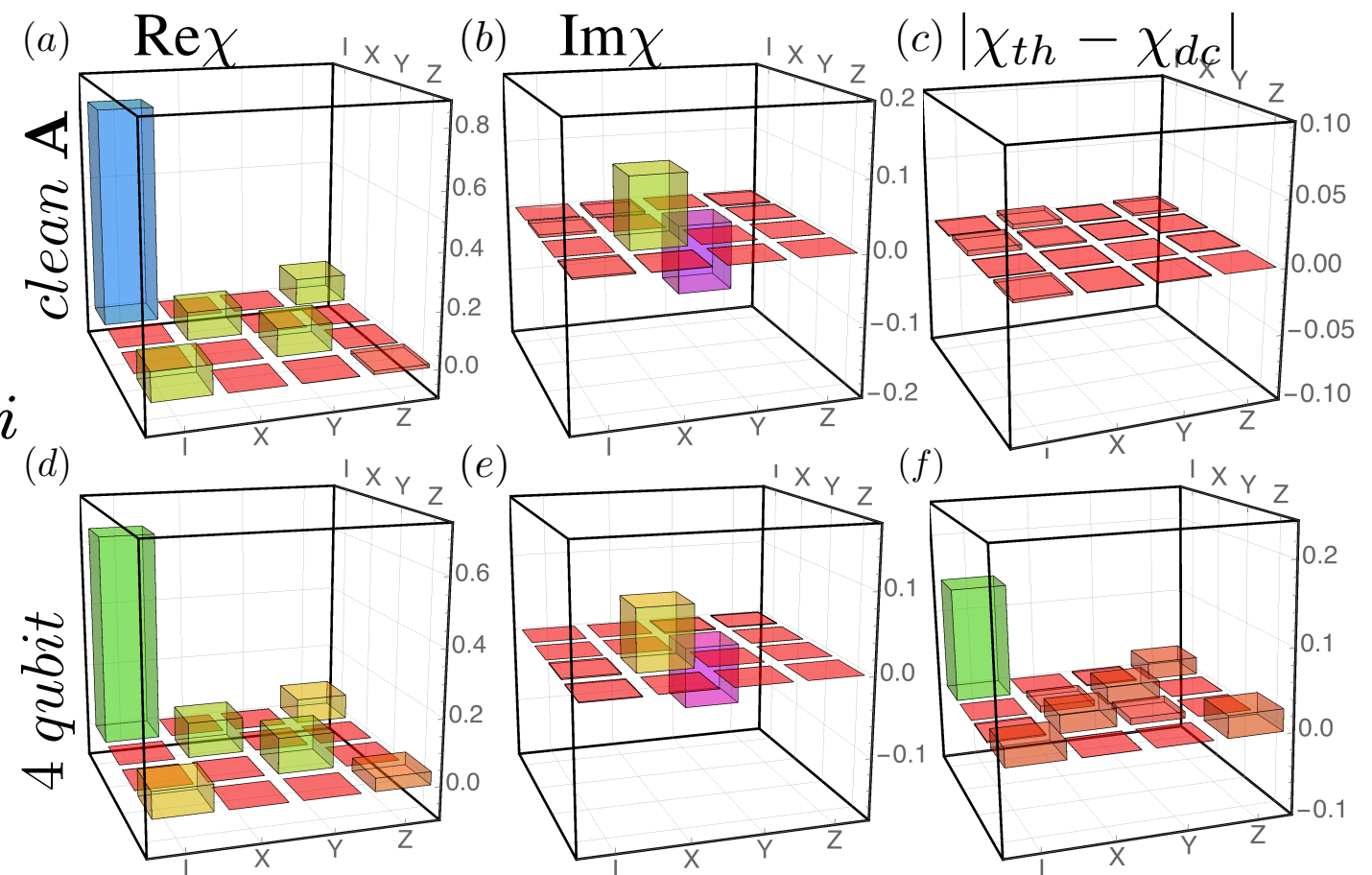
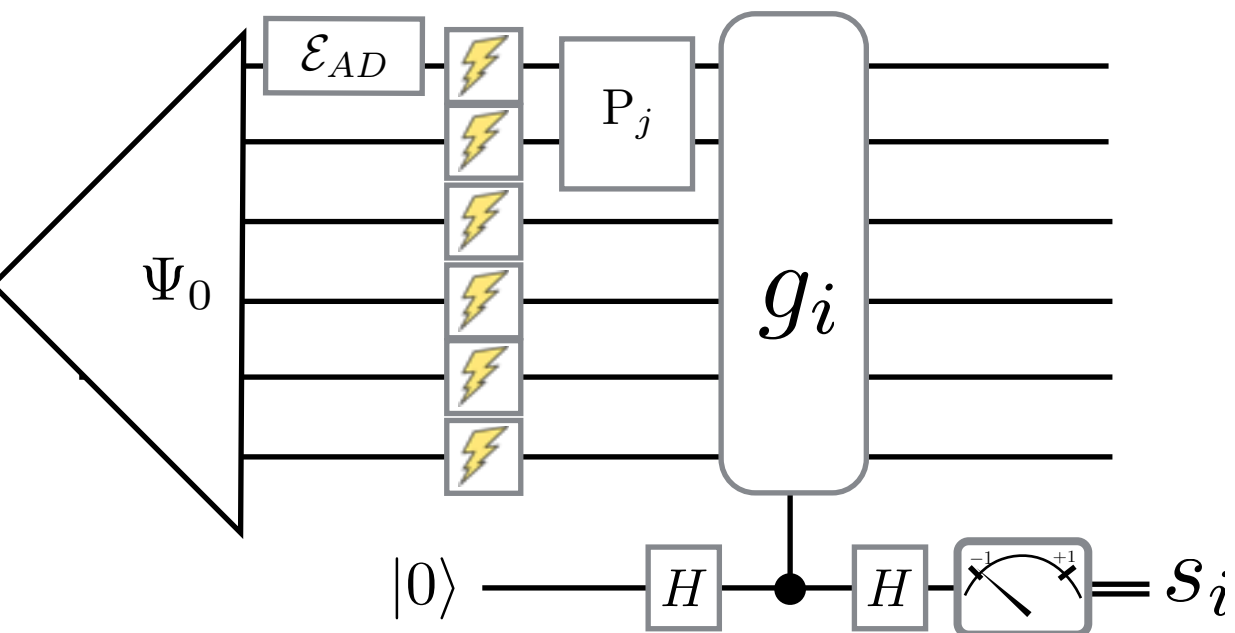


# Ex: Single Qubit Process

Depolarizing Noise  $p_{DP} = 0.1$

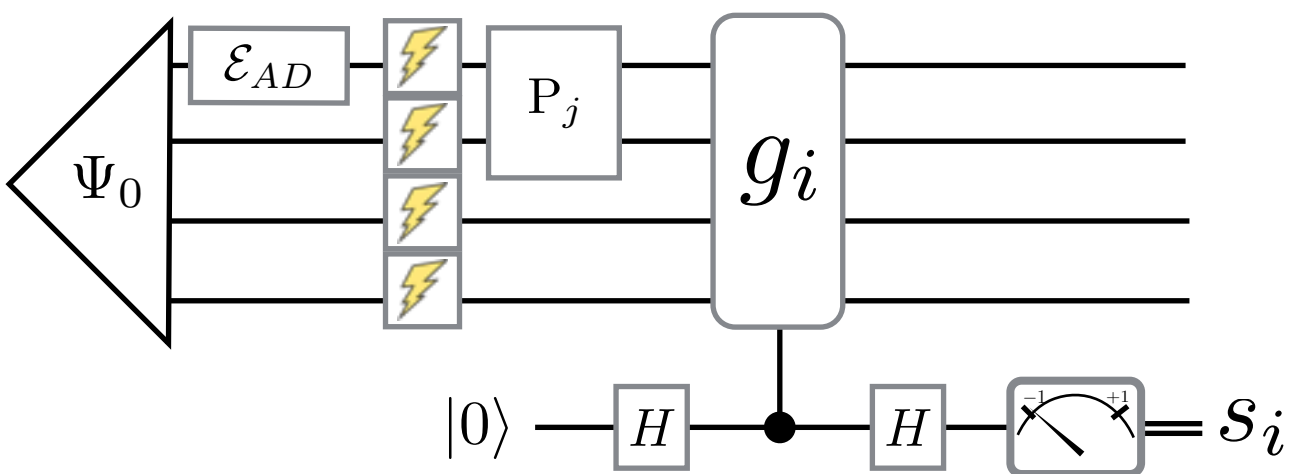


$$F \left[ \mathcal{E}^{AD}(\rho), \mathcal{E}^{[[4,0,2]]}(\rho) \right] = .9165$$

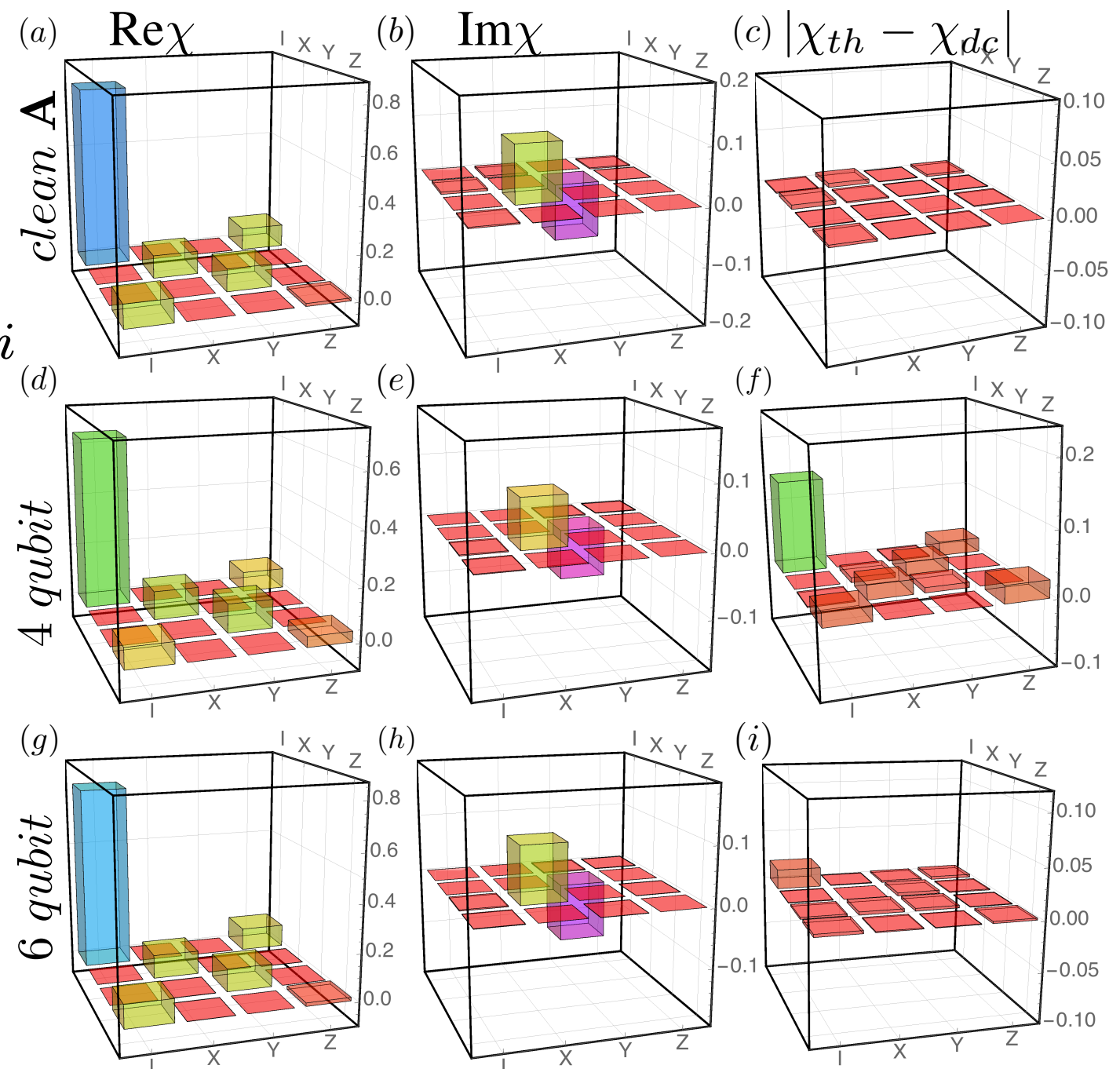
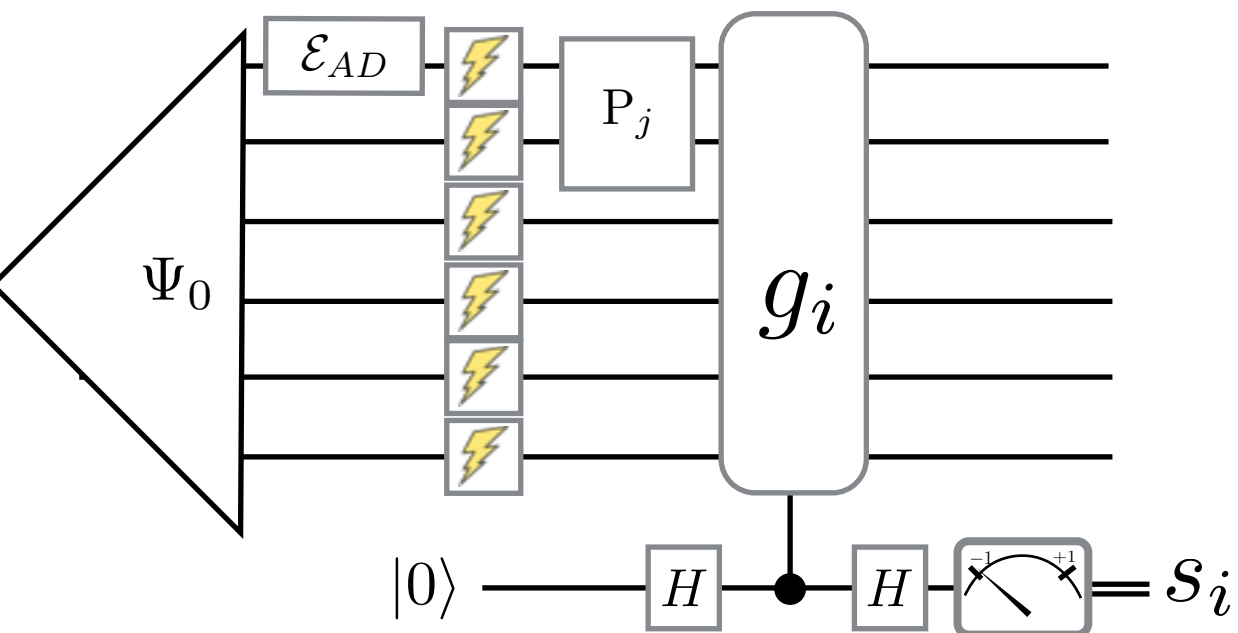


# Ex: Single Qubit Process

Depolarizing Noise  $p_{DP} = 0.1$



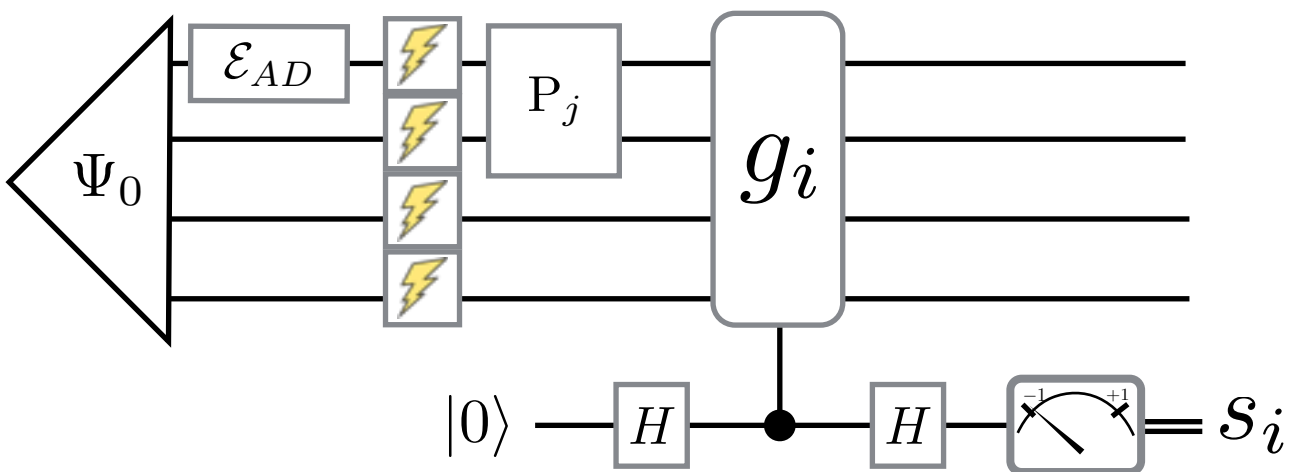
$$F \left[ \mathcal{E}^{AD}(\rho), \mathcal{E}^{[[4,0,2]]}(\rho) \right] = .9165$$



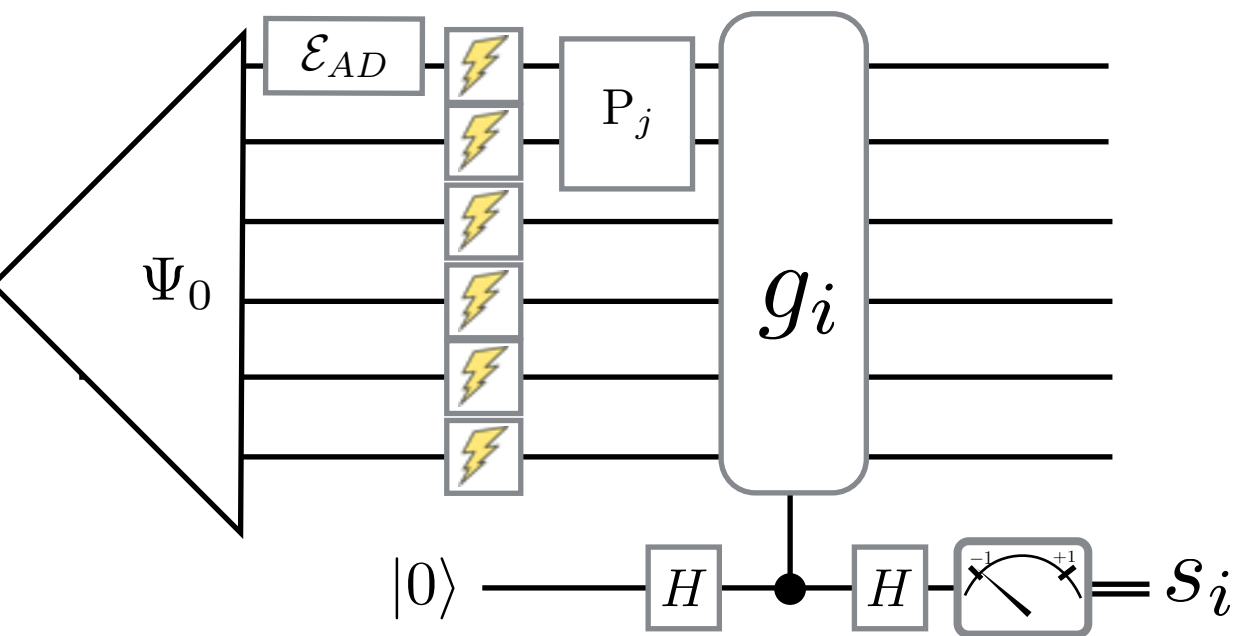


# Ex: Single Qubit Process

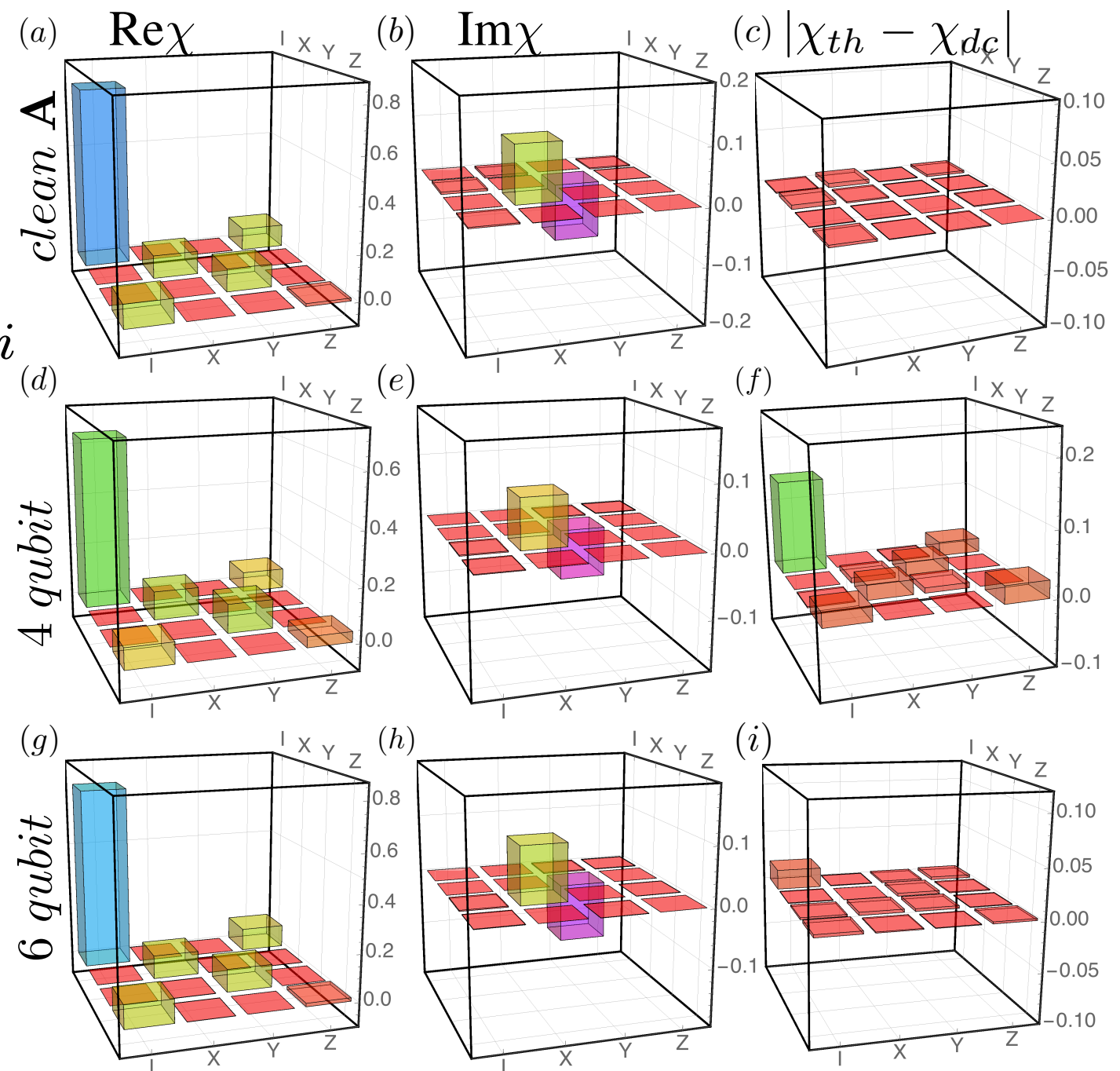
Depolarizing Noise  $p_{DP} = 0.1$



$$F \left[ \mathcal{E}^{AD}(\rho), \mathcal{E}^{[[4,0,2]]}(\rho) \right] = .9165$$



$$F \left[ \mathcal{E}^{AD}(\rho), \mathcal{E}^{[[6,0,2]]}(\rho) \right] = .9884$$

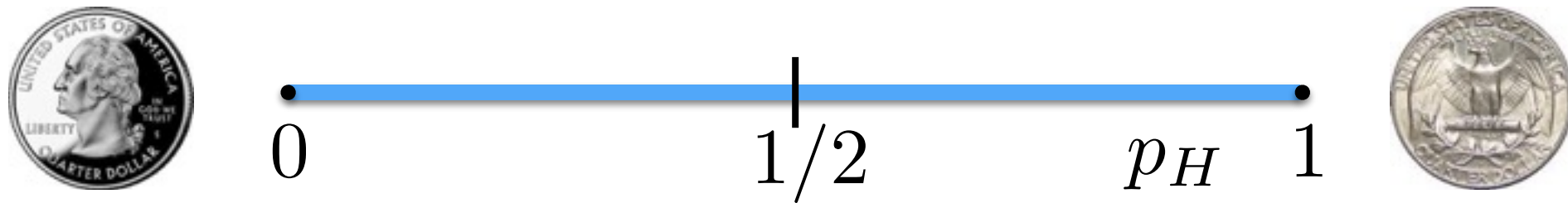


# Classical Model Selection



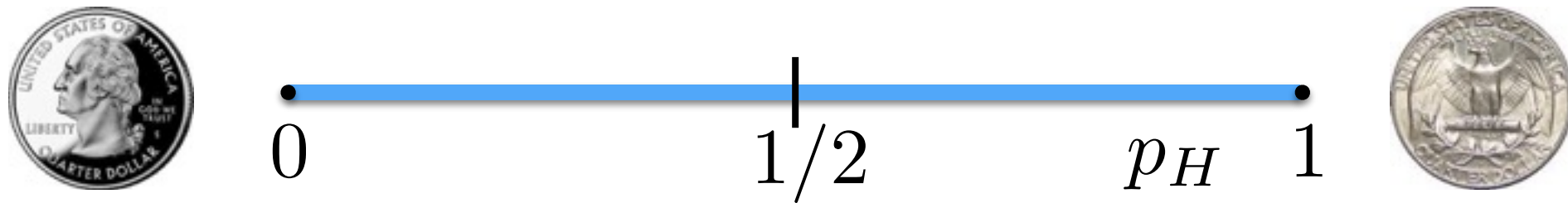
# Classical Model Selection

Q: Is coin fair?



# Classical Model Selection

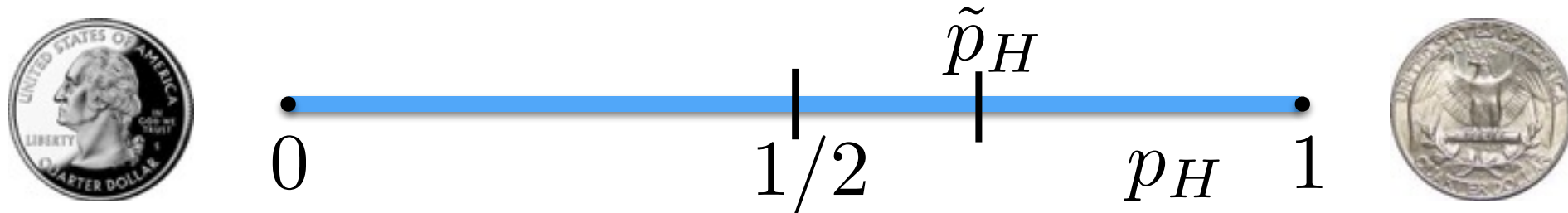
Q: Is coin fair?



$$X = \left\{ \begin{array}{c} \text{US Quarter} \\ \text{US Dime} \\ \text{US Dime} \\ \text{US Dime} \\ \text{US Dime} \\ \text{US Quarter} \\ \text{US Quarter} \\ \text{US Dime} \\ \dots \\ \text{US Quarter} \end{array} \right\}$$

# Classical Model Selection

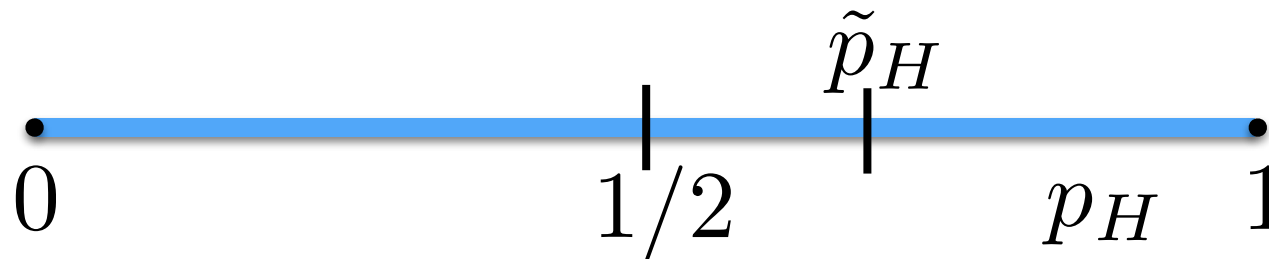
Q: Is coin fair?



$$X = \left\{ \begin{array}{c} \text{coin 1} \\ \text{coin 2} \\ \text{coin 3} \\ \text{coin 4} \\ \text{coin 5} \\ \text{coin 6} \\ \text{coin 7} \\ \dots \\ \text{coin } N_{tot} \end{array} \right\} \quad \tilde{p}_H = \frac{N_H}{N_{tot}}$$

# Classical Model Selection

Q: Is coin fair?



$$X = \left\{ \begin{array}{c} \text{Obverse} \\ \text{Reverse} \\ \text{Reverse} \\ \text{Reverse} \\ \text{Obverse} \\ \text{Obverse} \\ \text{Reverse} \\ \dots \\ \text{Obverse} \end{array} \right\} \quad \tilde{p}_H = \frac{N_H}{N_{tot}}$$

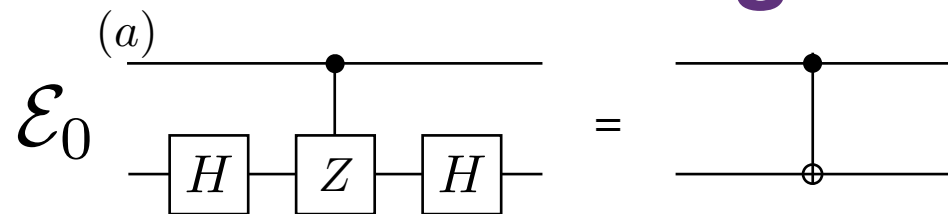
$$\text{Log} \left( \frac{\mathcal{L}_{alt}}{\mathcal{L}_{null}} = \frac{\text{Prob}(X|p_H = 1/2)}{\text{Prob}(X|p_H = \tilde{p}_H)} \right) \begin{cases} < D^* & \text{null} \\ \geq D^* & \text{alternative} \end{cases}$$

Choice:

# Quantum Channel Discrimination

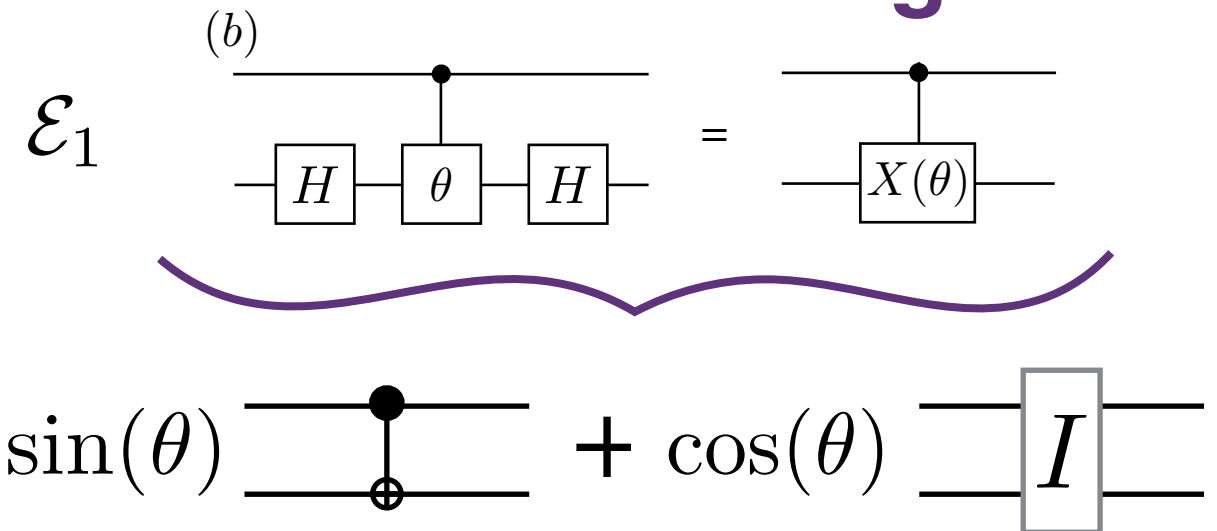
# Quantum Channel Discrimination

**Perfect Entangler:**



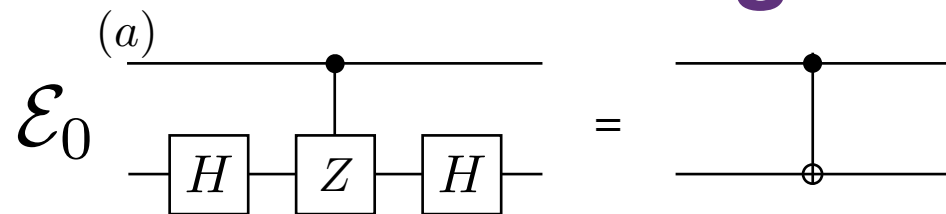
**vs.**

**Partial Entangler:**



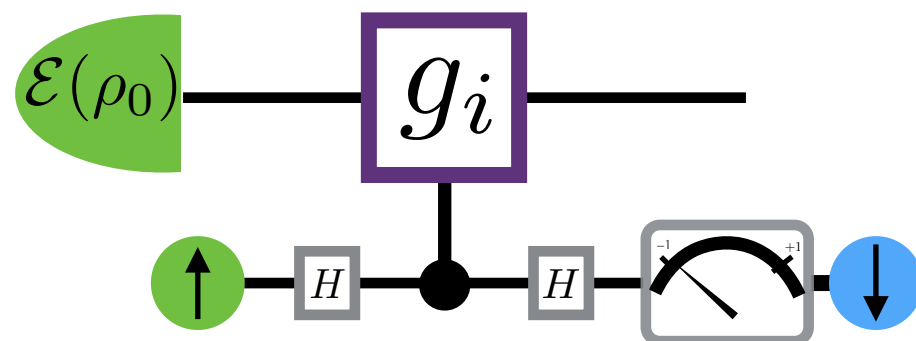
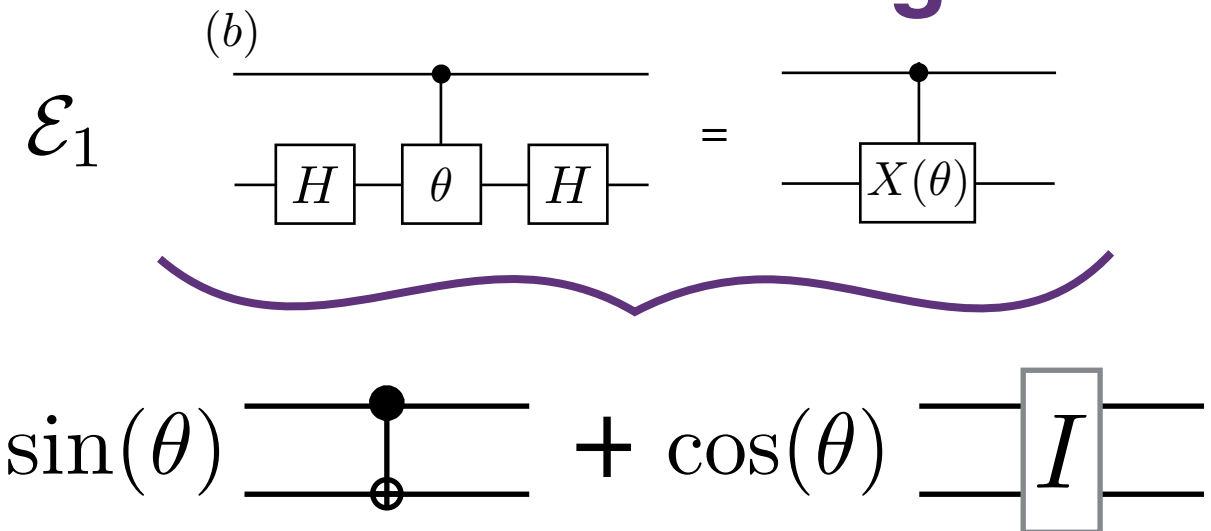
# Quantum Channel Discrimination

## Perfect Entangler:



vs.

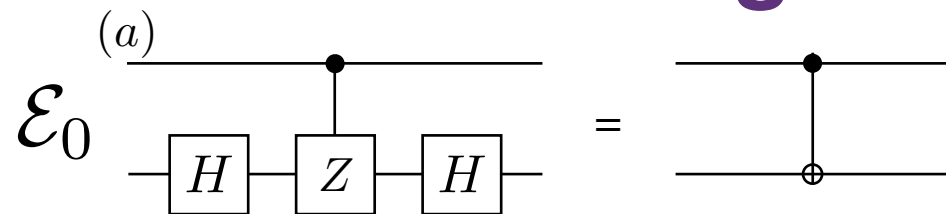
## Partial Entangler:



$$\chi^{(CX_{12}(\theta))} = \frac{1}{8} \begin{pmatrix} \mathbb{1}\mathbb{1} & Z\mathbb{1} & \mathbb{1}X & ZX \\ \begin{matrix} 3\cos(\theta) + 5 \\ -\cos(\theta) - 2i\sin(\theta) + 1 \\ -\cos(\theta) - 2i\sin(\theta) + 1 \\ \cos(\theta) + 2i\sin(\theta) - 1 \end{matrix} & \begin{matrix} -\cos(\theta) + 2i\sin(\theta) + 1 \\ 1 - \cos(\theta) \\ 1 - \cos(\theta) \\ \cos(\theta) - 1 \end{matrix} & \begin{matrix} -\cos(\theta) + 2i\sin(\theta) + 1 \\ 1 - \cos(\theta) \\ 1 - \cos(\theta) \\ \cos(\theta) - 1 \end{matrix} & \begin{matrix} \cos(\theta) - 2i\sin(\theta) - 1 \\ \cos(\theta) - 1 \\ \cos(\theta) - 1 \\ 1 - \cos(\theta) \end{matrix} \end{pmatrix} \begin{matrix} \mathbb{1}\mathbb{1} \\ Z\mathbb{1} \\ \mathbb{1}X \\ ZX \end{matrix}$$

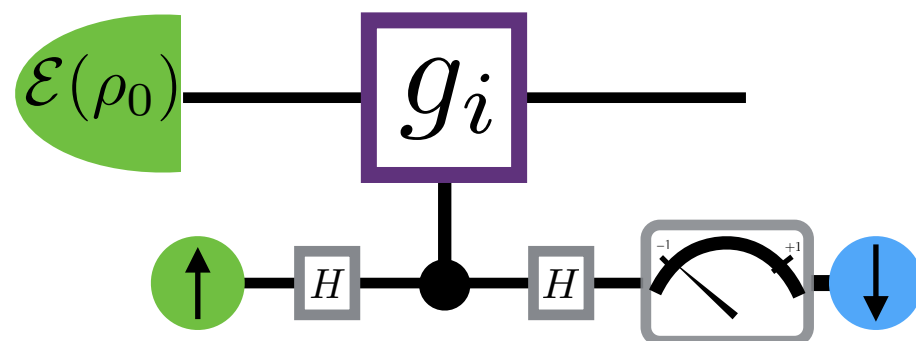
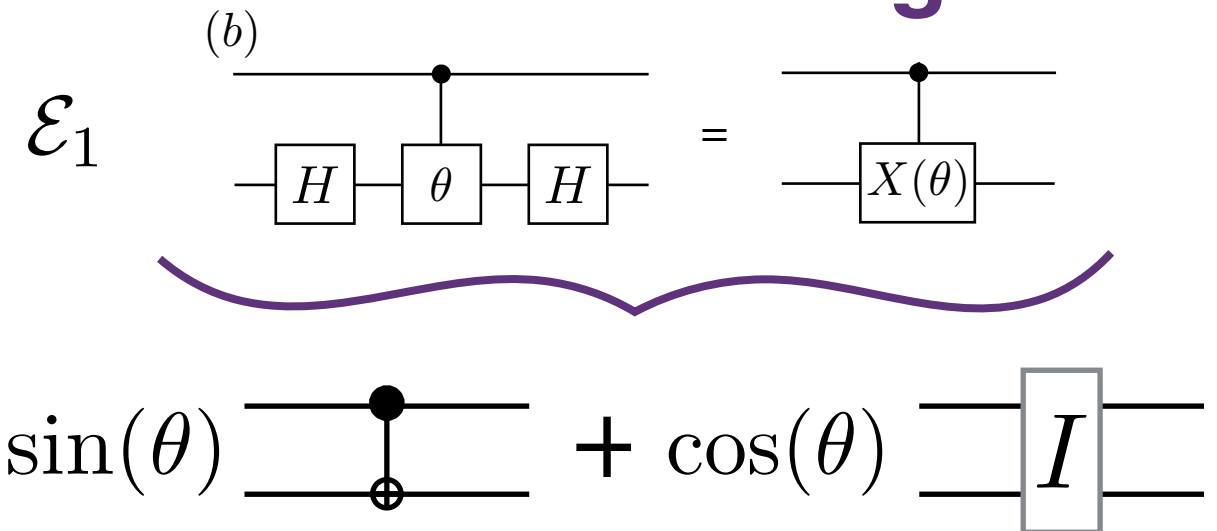
# Quantum Channel Discrimination

## Perfect Entangler:



vs.

## Partial Entangler:



$$\chi^{(CX_{12}(\theta))} = \frac{1}{8} \begin{pmatrix} \mathbb{1}\mathbb{1} & Z\mathbb{1} & \mathbb{1}X & ZX \\ \begin{matrix} 3\cos(\theta) + 5 \\ -\cos(\theta) - 2i\sin(\theta) + 1 \\ -\cos(\theta) - 2i\sin(\theta) + 1 \\ \cos(\theta) + 2i\sin(\theta) - 1 \end{matrix} & \begin{matrix} -\cos(\theta) + 2i\sin(\theta) + 1 \\ 1 - \cos(\theta) \\ 1 - \cos(\theta) \\ \cos(\theta) - 1 \end{matrix} & \begin{matrix} -\cos(\theta) + 2i\sin(\theta) + 1 \\ 1 - \cos(\theta) \\ 1 - \cos(\theta) \\ \cos(\theta) - 1 \end{matrix} & \begin{matrix} \cos(\theta) - 2i\sin(\theta) - 1 \\ \cos(\theta) - 1 \\ \cos(\theta) - 1 \\ 1 - \cos(\theta) \end{matrix} \end{pmatrix} \begin{matrix} \mathbb{1}\mathbb{1} \\ Z\mathbb{1} \\ \mathbb{1}X \\ ZX \end{matrix}$$

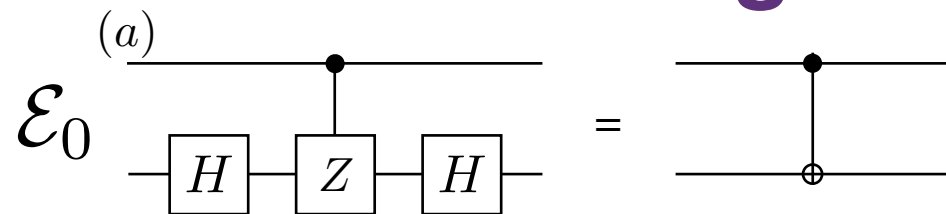
## Procedure:

1. Perform judicious stabilizer measurements  $\hat{\chi}_{ij}$



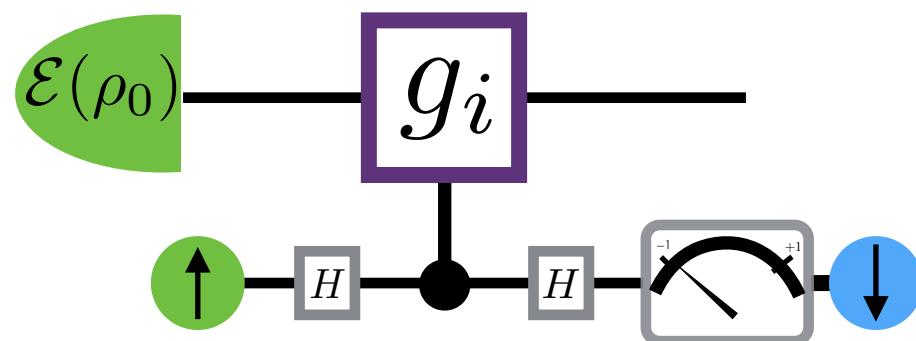
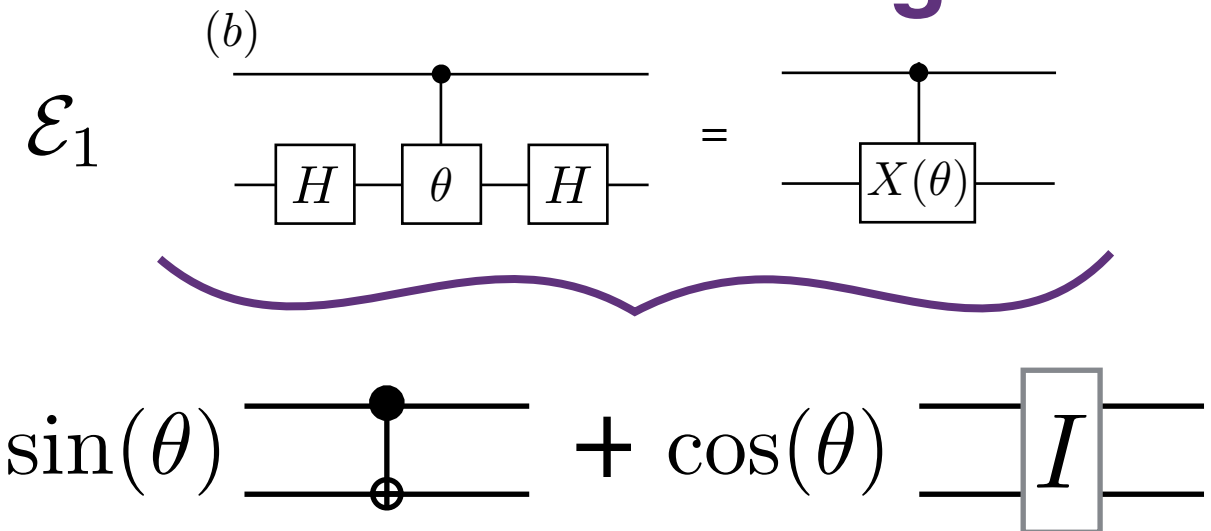
# Quantum Channel Discrimination

## Perfect Entangler:



vs.

## Partial Entangler:



$$\chi^{(CX_{12}(\theta))} = \frac{1}{8} \begin{pmatrix} \mathbb{1}\mathbb{1} & Z\mathbb{1} & \mathbb{1}X & ZX \\ \begin{matrix} 3\cos(\theta) + 5 \\ -\cos(\theta) - 2i\sin(\theta) + 1 \\ -\cos(\theta) - 2i\sin(\theta) + 1 \\ \cos(\theta) + 2i\sin(\theta) - 1 \end{matrix} & \begin{matrix} -\cos(\theta) + 2i\sin(\theta) + 1 \\ 1 - \cos(\theta) \\ 1 - \cos(\theta) \\ \cos(\theta) - 1 \end{matrix} & \begin{matrix} -\cos(\theta) + 2i\sin(\theta) + 1 \\ 1 - \cos(\theta) \\ 1 - \cos(\theta) \\ \cos(\theta) - 1 \end{matrix} & \begin{matrix} \cos(\theta) - 2i\sin(\theta) - 1 \\ \cos(\theta) - 1 \\ \cos(\theta) - 1 \\ 1 - \cos(\theta) \end{matrix} \end{pmatrix} \begin{matrix} \mathbb{1}\mathbb{1} \\ Z\mathbb{1} \\ \mathbb{1}X \\ ZX \end{matrix}$$

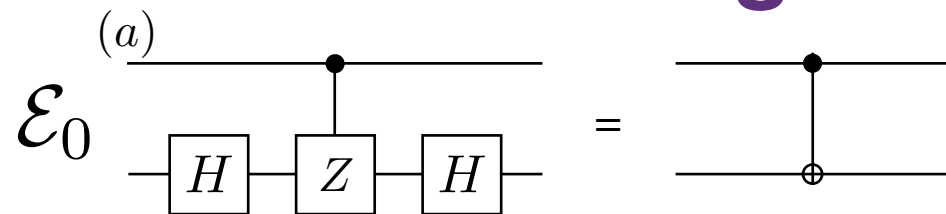
## Procedure:

1. Perform judicious stabilizer measurements
2. Estimate channel parameter

$$\hat{\chi}_{ij} \rightarrow \hat{\theta}$$

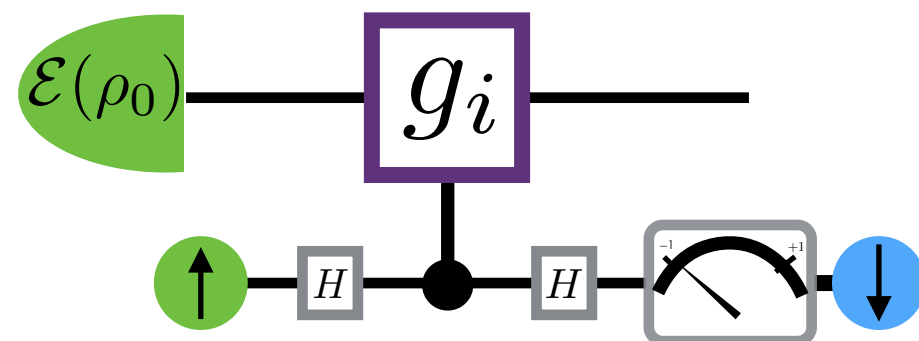
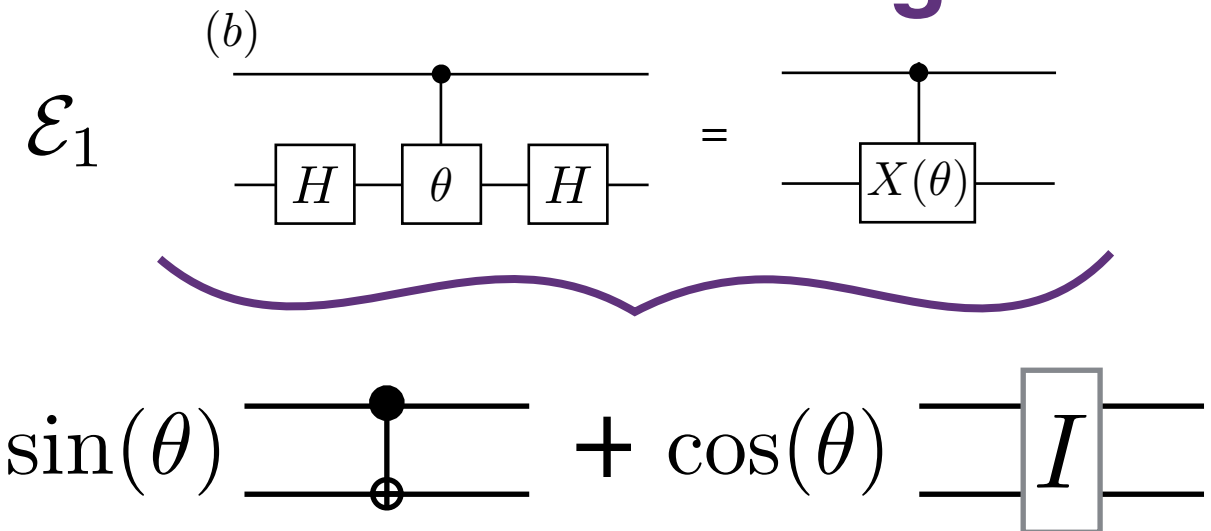
# Quantum Channel Discrimination

## Perfect Entangler:



vs.

## Partial Entangler:



$$\chi^{(CX_{12}(\theta))} = \frac{1}{8} \begin{pmatrix} \mathbb{1}\mathbb{1} & Z\mathbb{1} & \mathbb{1}X & ZX \\ \begin{matrix} 3\cos(\theta) + 5 \\ -\cos(\theta) - 2i\sin(\theta) + 1 \\ -\cos(\theta) - 2i\sin(\theta) + 1 \\ \cos(\theta) + 2i\sin(\theta) - 1 \end{matrix} & \begin{matrix} -\cos(\theta) + 2i\sin(\theta) + 1 \\ 1 - \cos(\theta) \\ 1 - \cos(\theta) \\ \cos(\theta) - 1 \end{matrix} & \begin{matrix} -\cos(\theta) + 2i\sin(\theta) + 1 \\ 1 - \cos(\theta) \\ 1 - \cos(\theta) \\ \cos(\theta) - 1 \end{matrix} & \begin{matrix} \cos(\theta) - 2i\sin(\theta) - 1 \\ \cos(\theta) - 1 \\ \cos(\theta) - 1 \\ 1 - \cos(\theta) \end{matrix} \end{pmatrix} \begin{matrix} \mathbb{1}\mathbb{1} \\ Z\mathbb{1} \\ \mathbb{1}X \\ ZX \end{matrix}$$

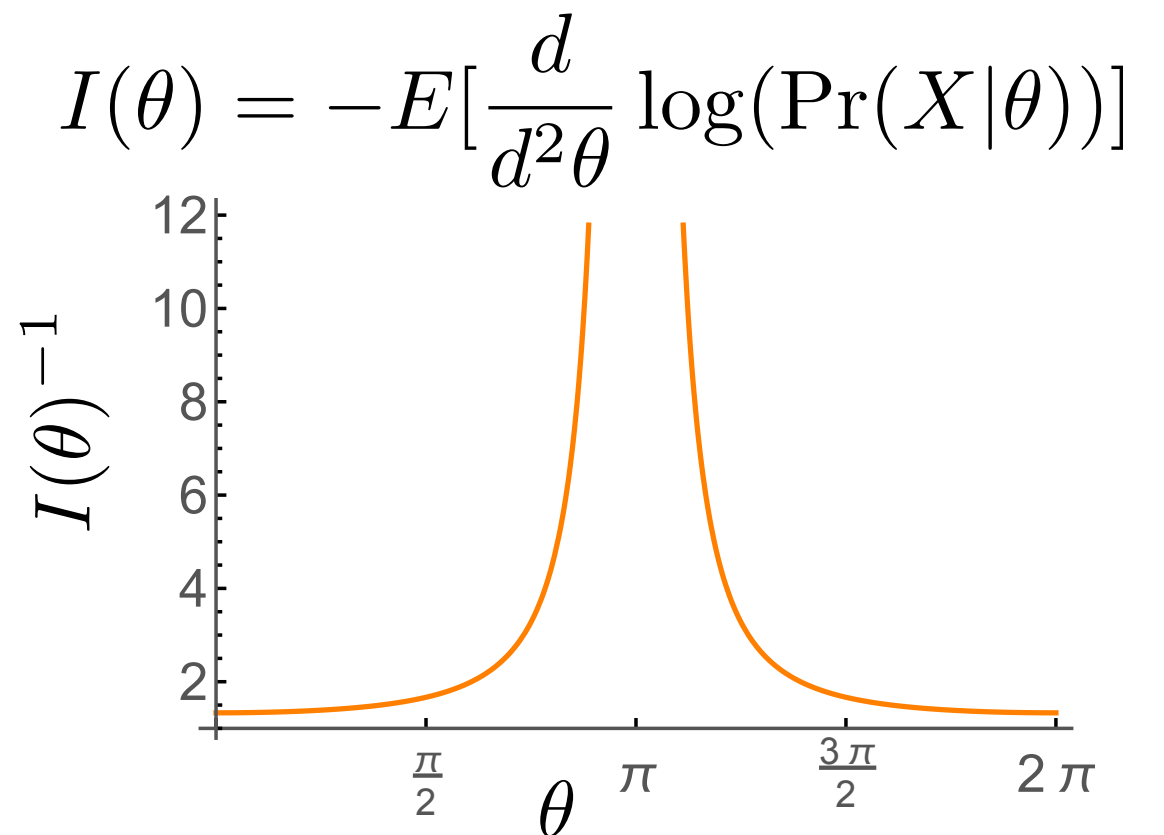
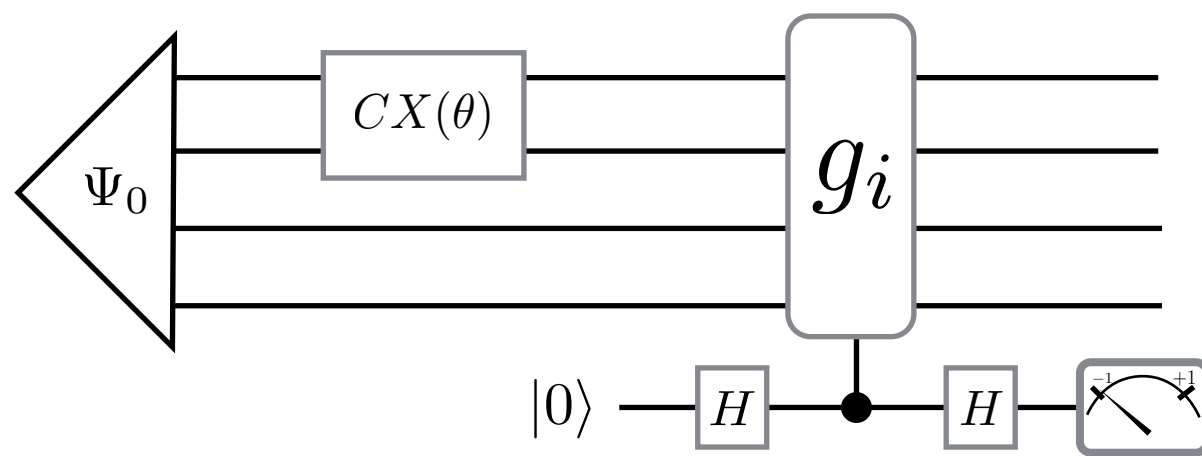
## Procedure:

1. Perform judicious stabilizer measurements
2. Estimate channel parameter
3. Statistical infer channel character

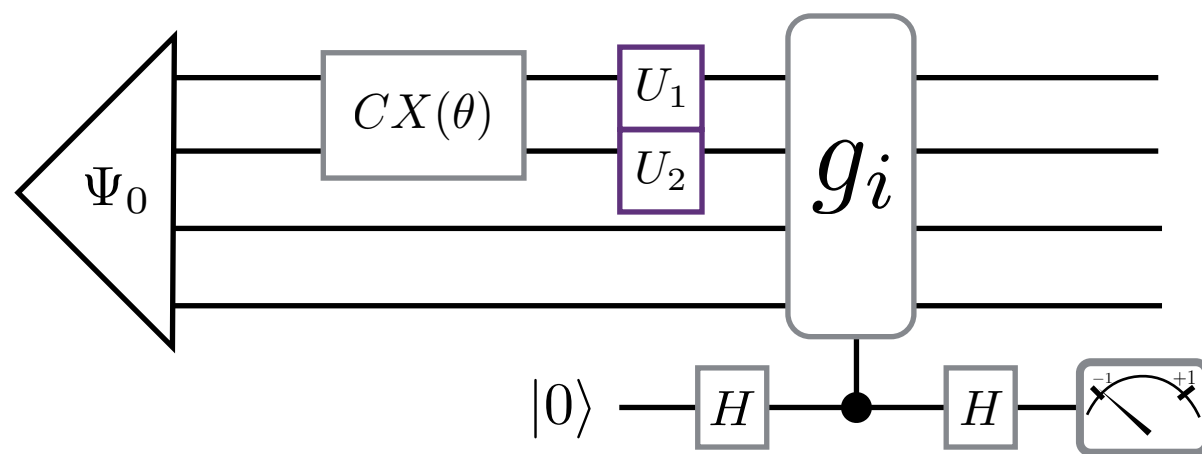
$$\begin{aligned} & \hat{\chi}_{ij} \\ & \hat{\chi}(\theta) \rightarrow \hat{\theta} \\ & W(\hat{\theta}) \geq \lambda^* \end{aligned}$$

# Statistical Inference

# Statistical Inference



# Statistical Inference

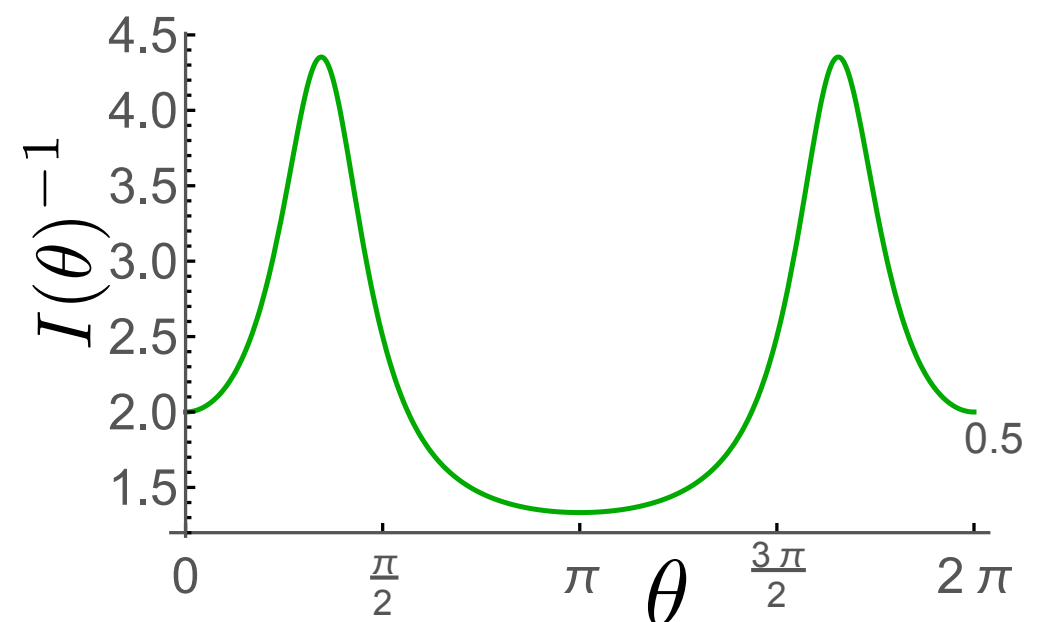
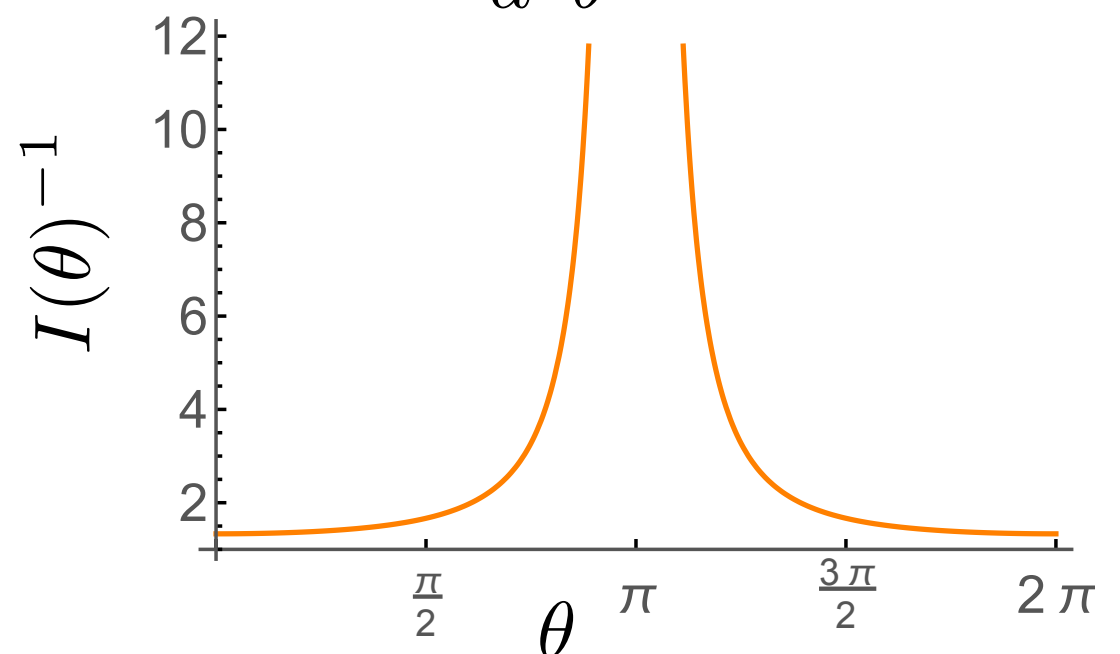


$$U_1 = (\mathbb{1} + iZ_1)/\sqrt{2}$$

$$U_2 = (\mathbb{1} + iX_2)/\sqrt{2}$$

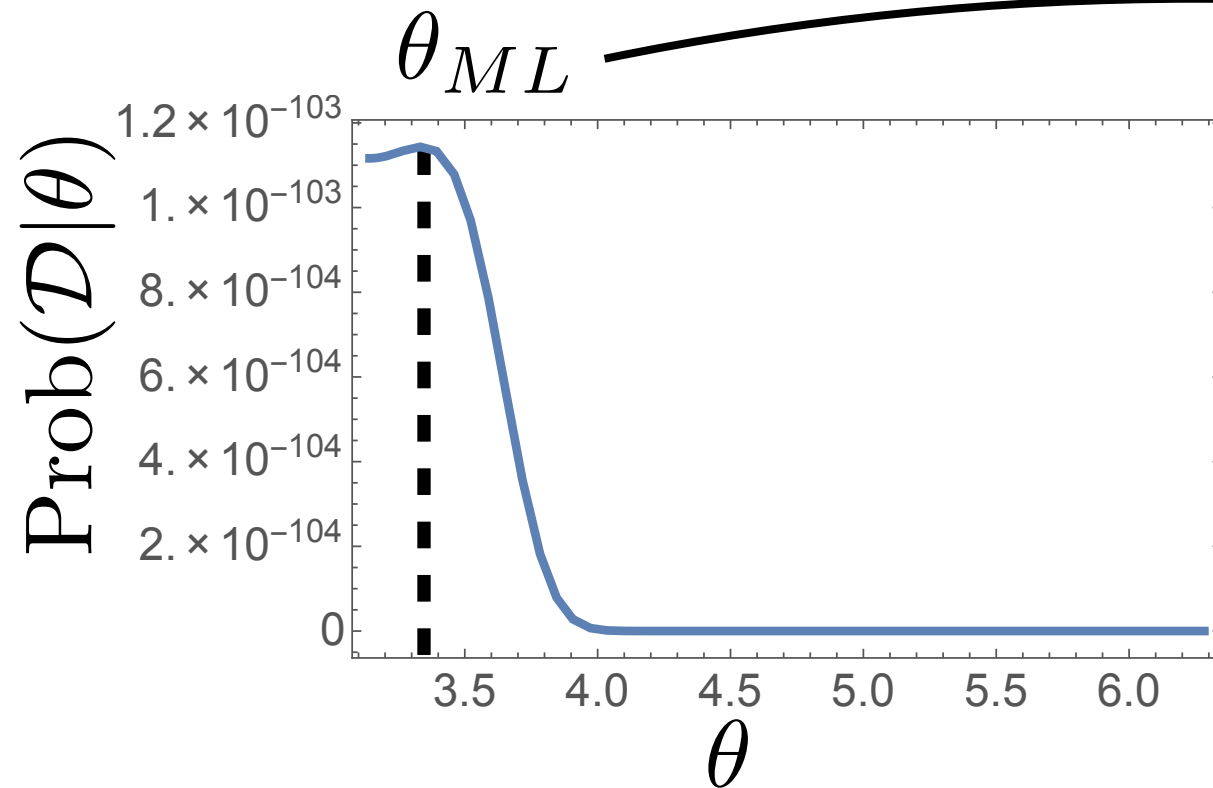
Switch to new basis with better CRLB

$$I(\theta) = -E\left[\frac{d}{d^2\theta} \log(\text{Pr}(X|\theta))\right]$$



# Statistical Inference

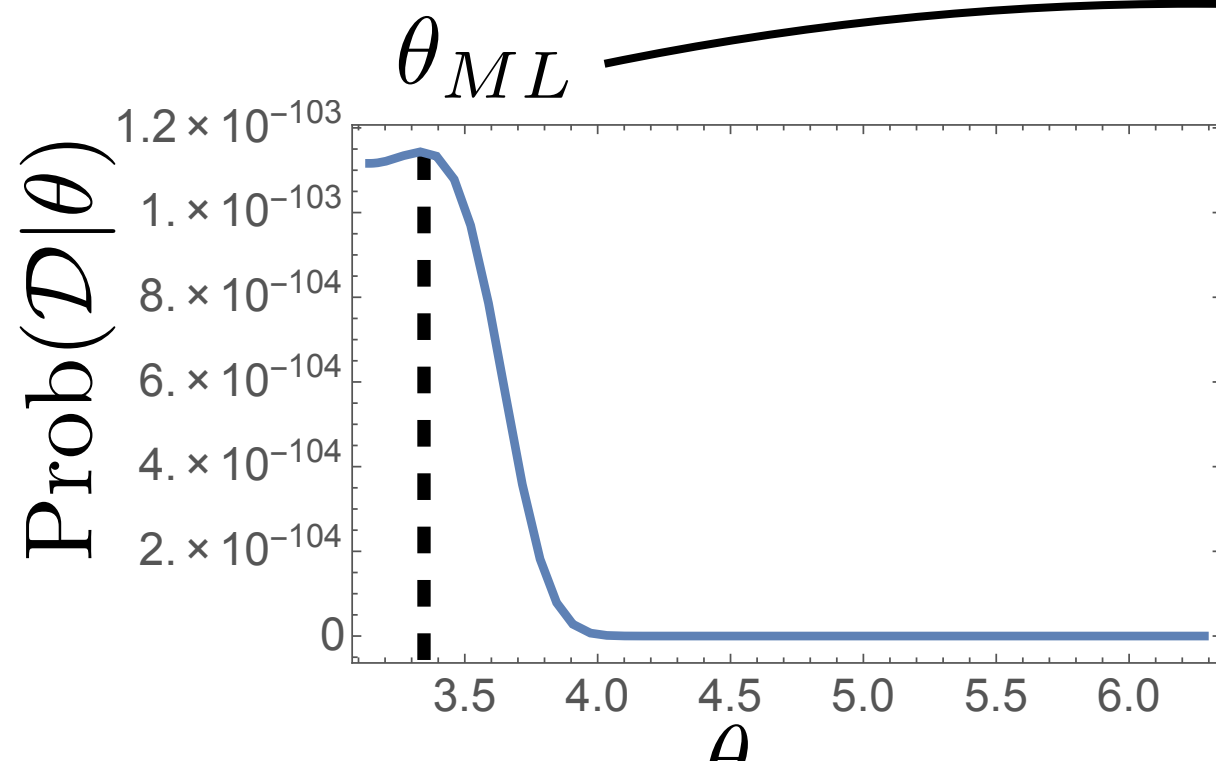
# Statistical Inference



$$W = \frac{(\hat{\theta} - \theta_0)^2}{\text{Var}(\hat{\theta})}$$

**Wald Statistic**  
for channel discrimination

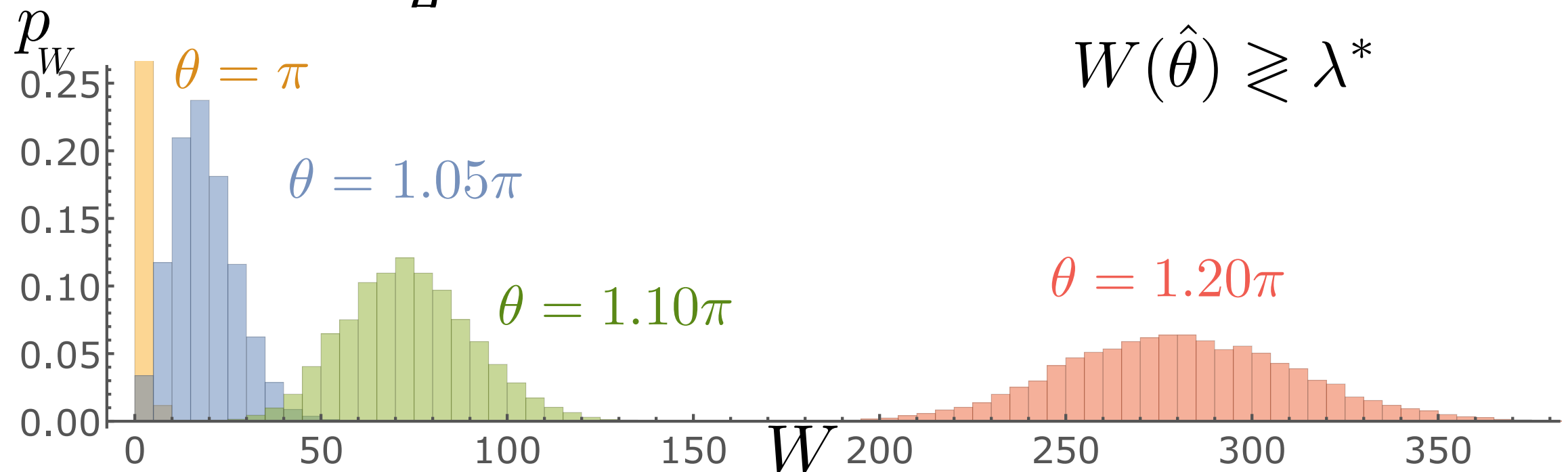
# Statistical Inference



$$W = \frac{(\hat{\theta} - \theta_0)^2}{\text{Var}(\hat{\theta})}$$

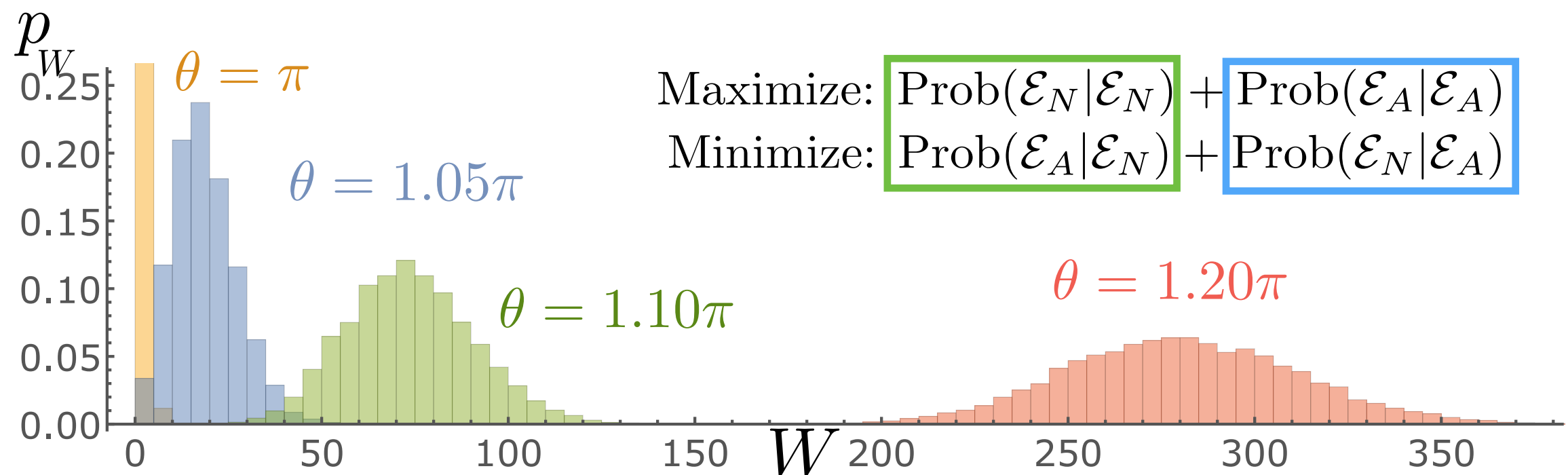
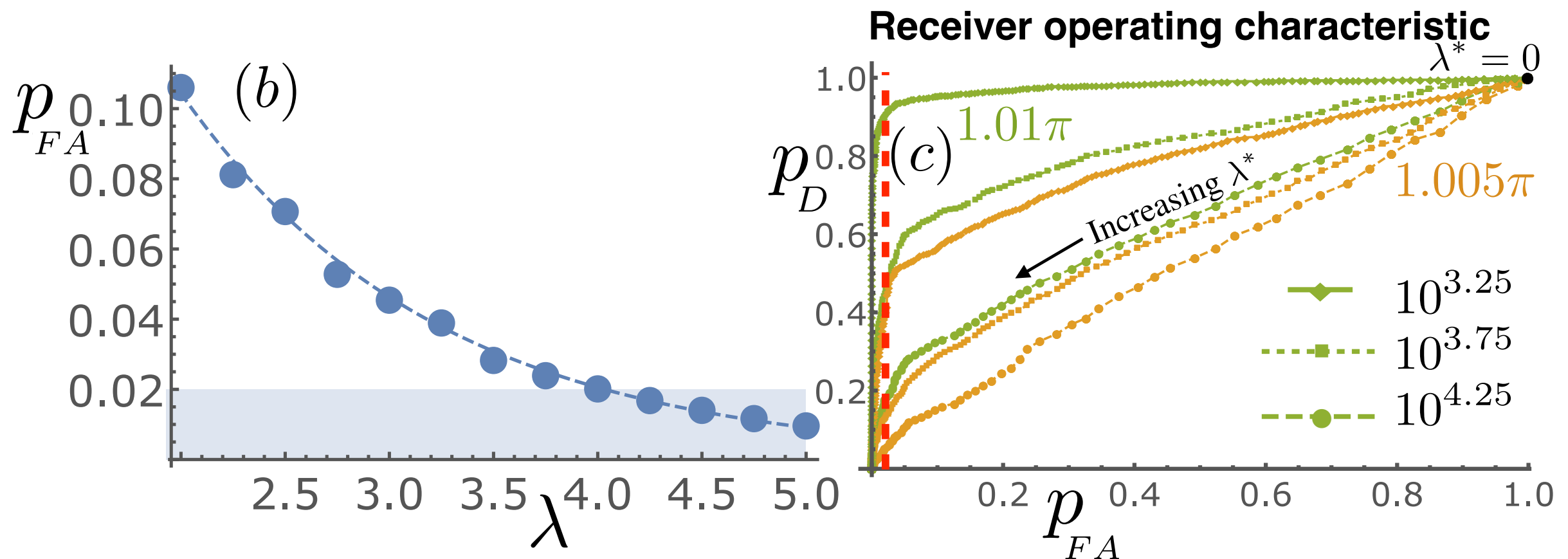
**Wald Statistic**  
for channel discrimination

$$W(\hat{\theta}) \geq \lambda^*$$



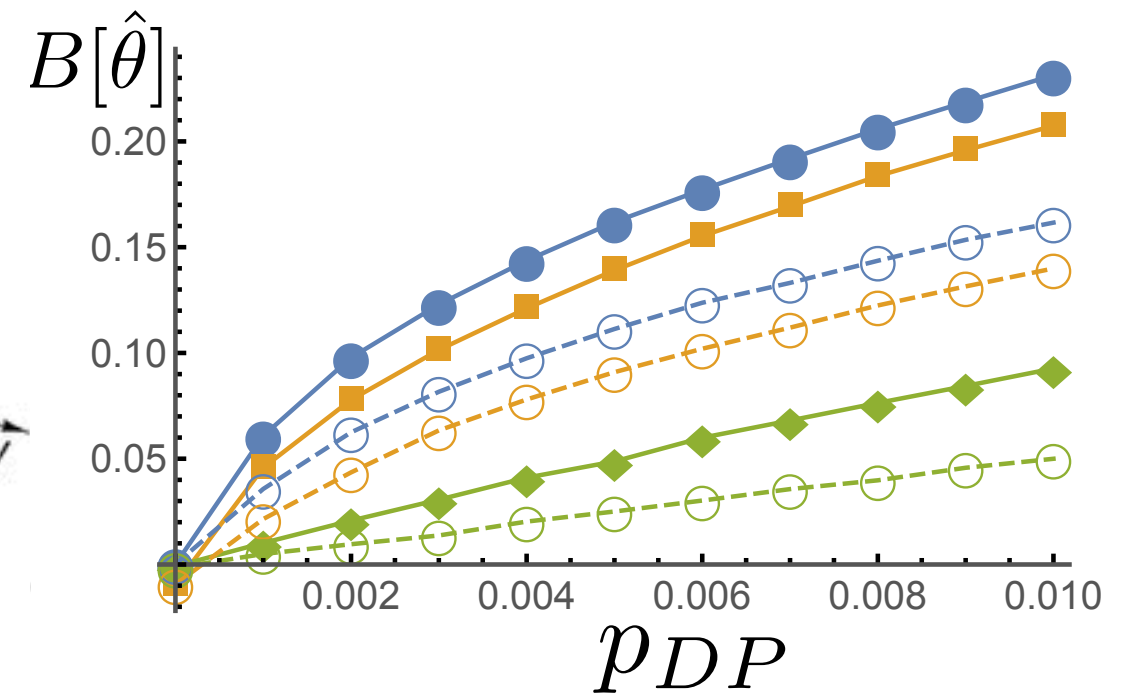
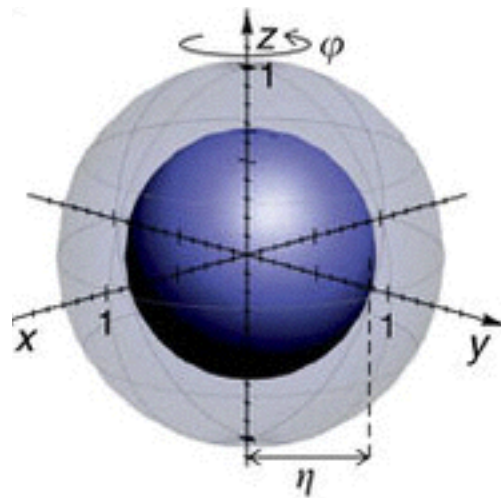
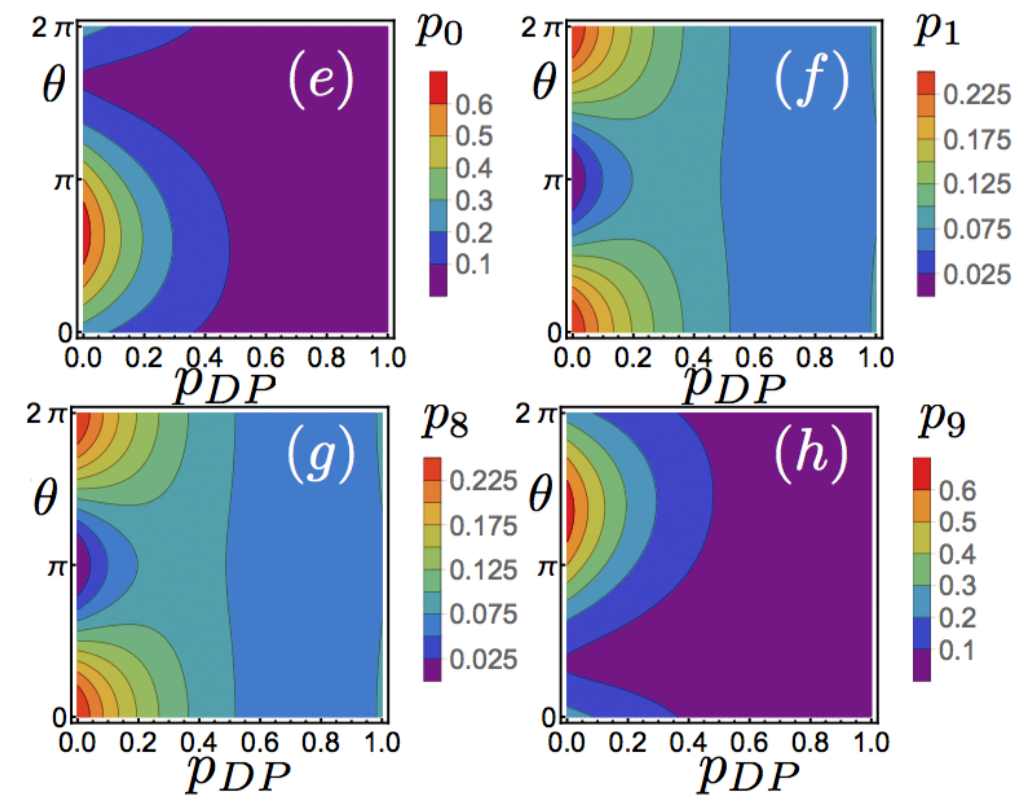


# Statistical Inference

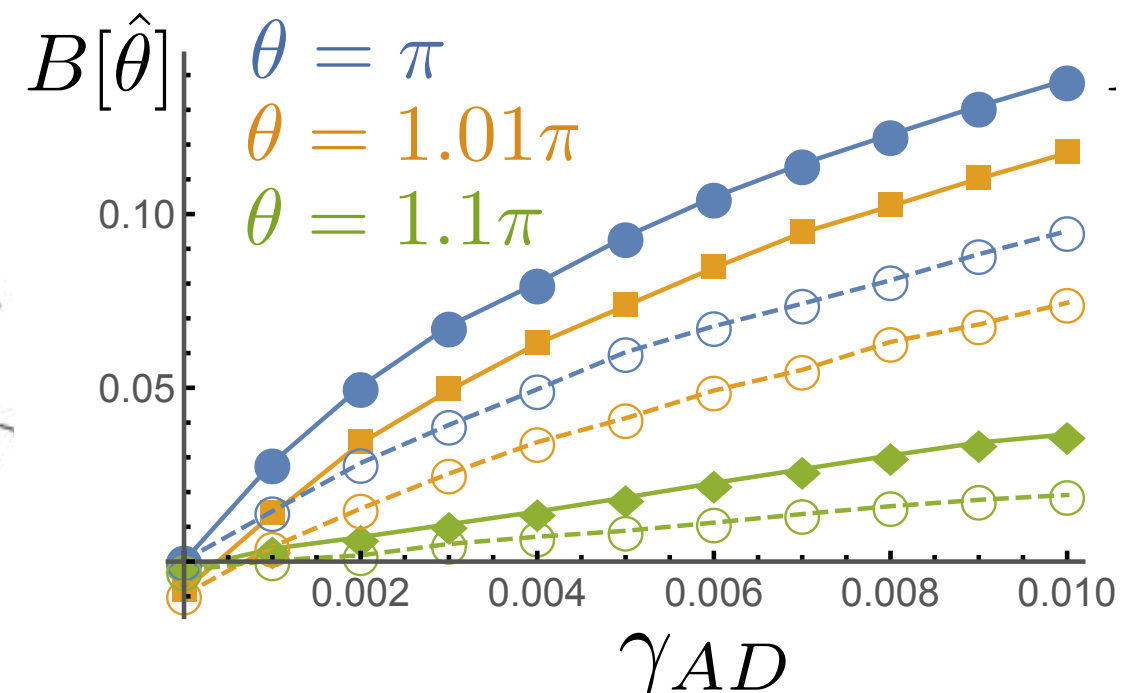
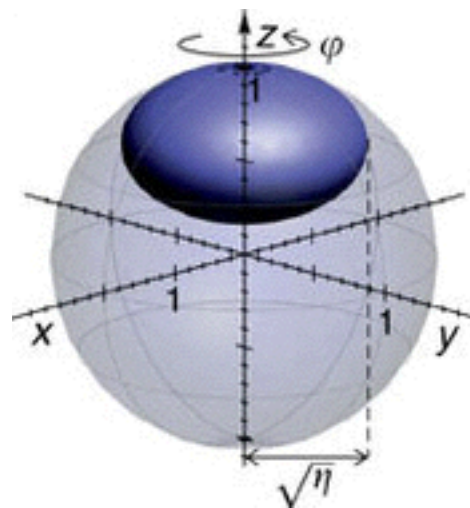
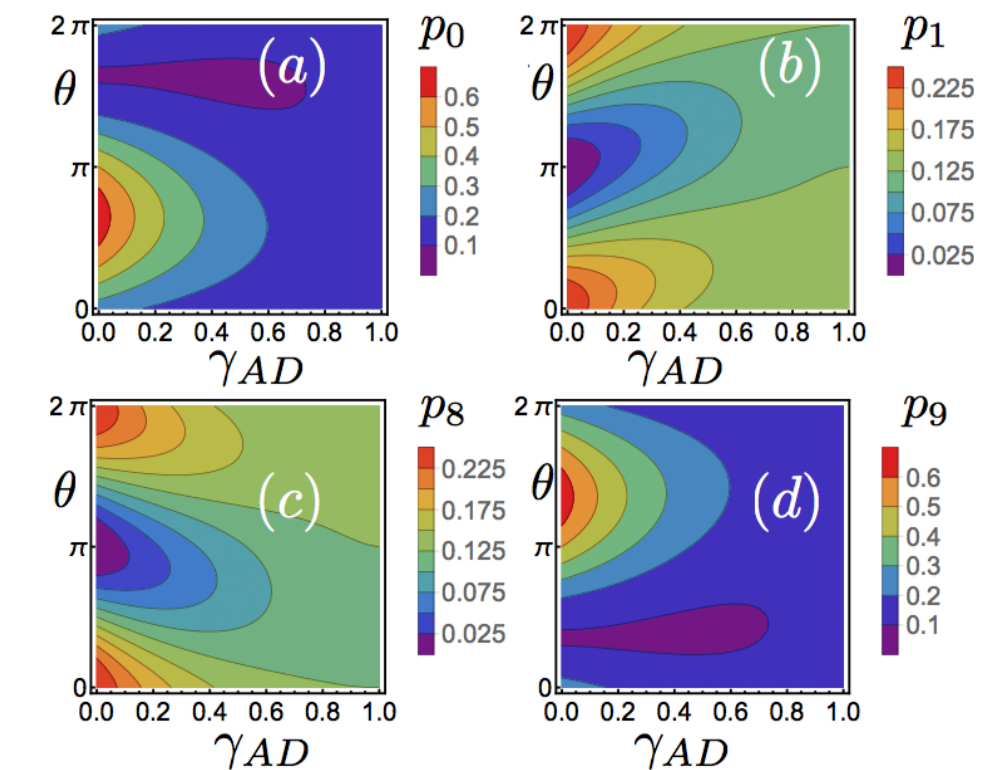
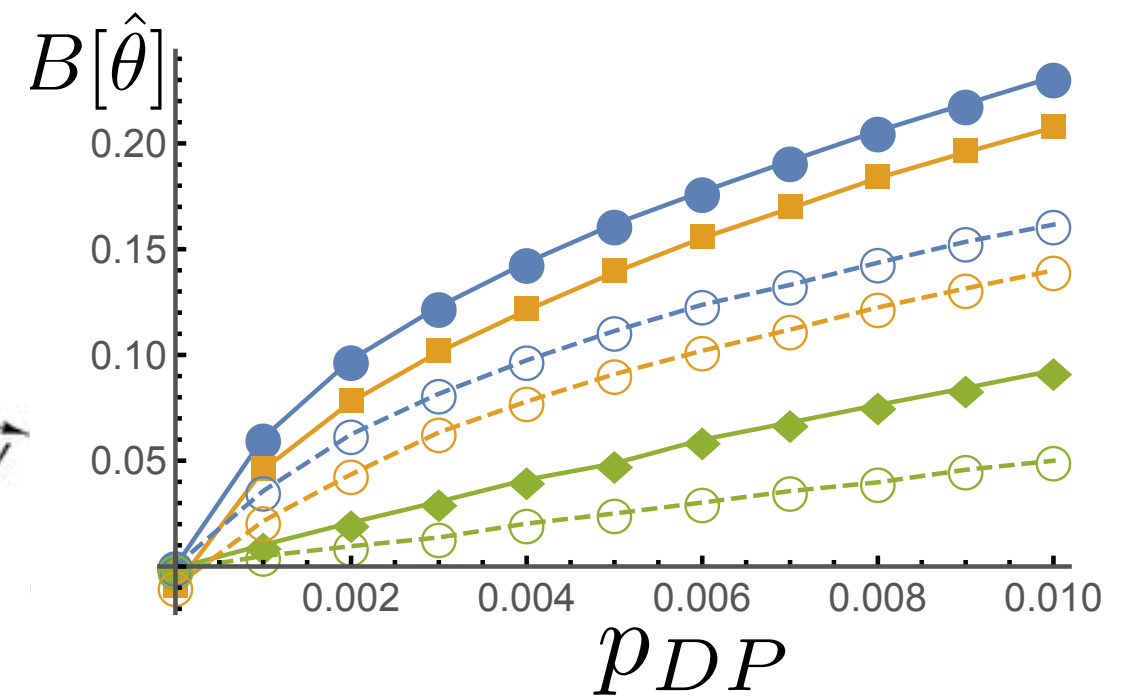
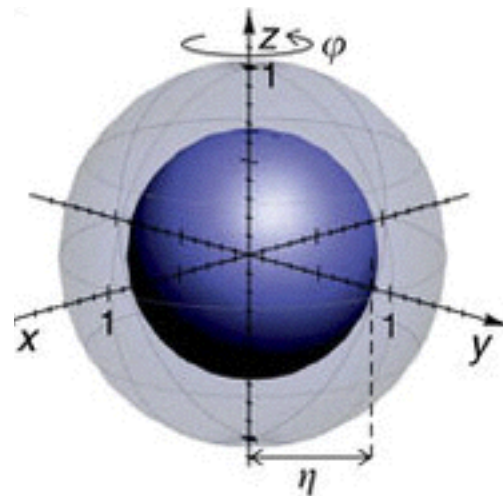
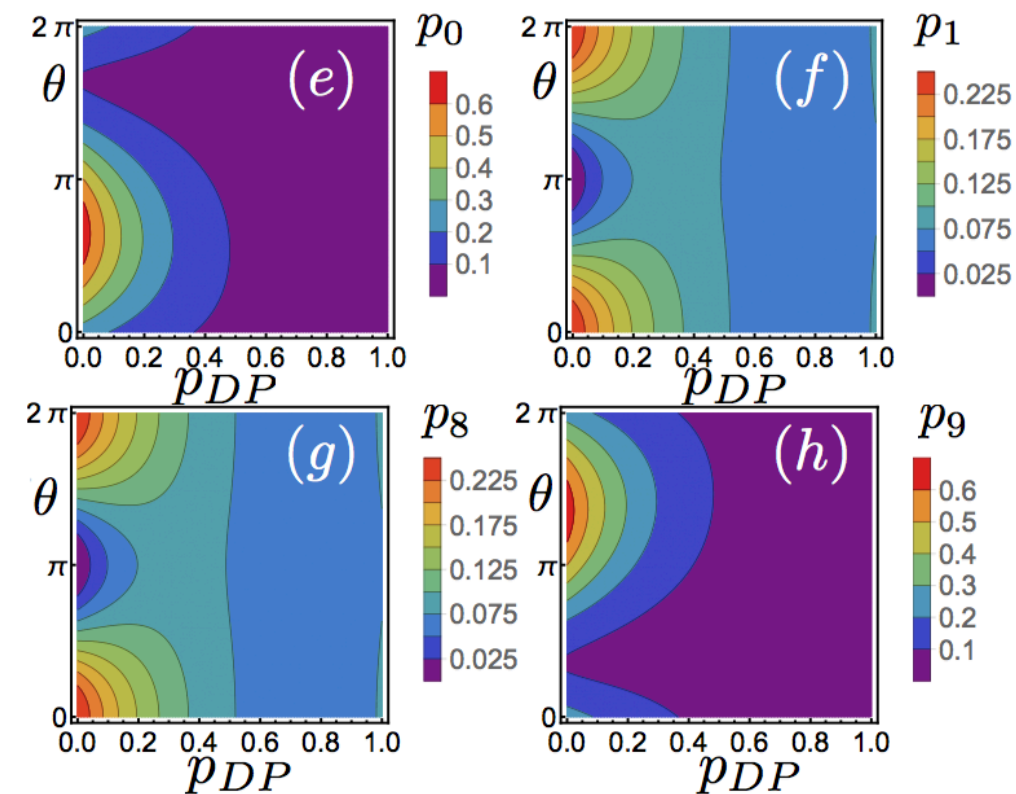


# Noisy parity checks

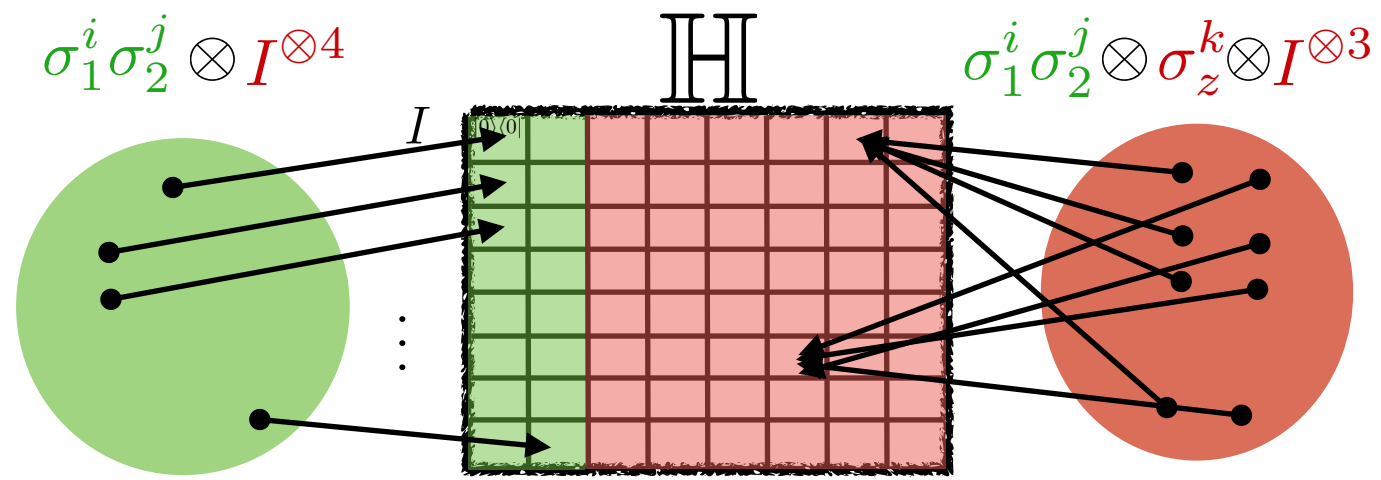
# Noisy parity checks



# Noisy parity checks

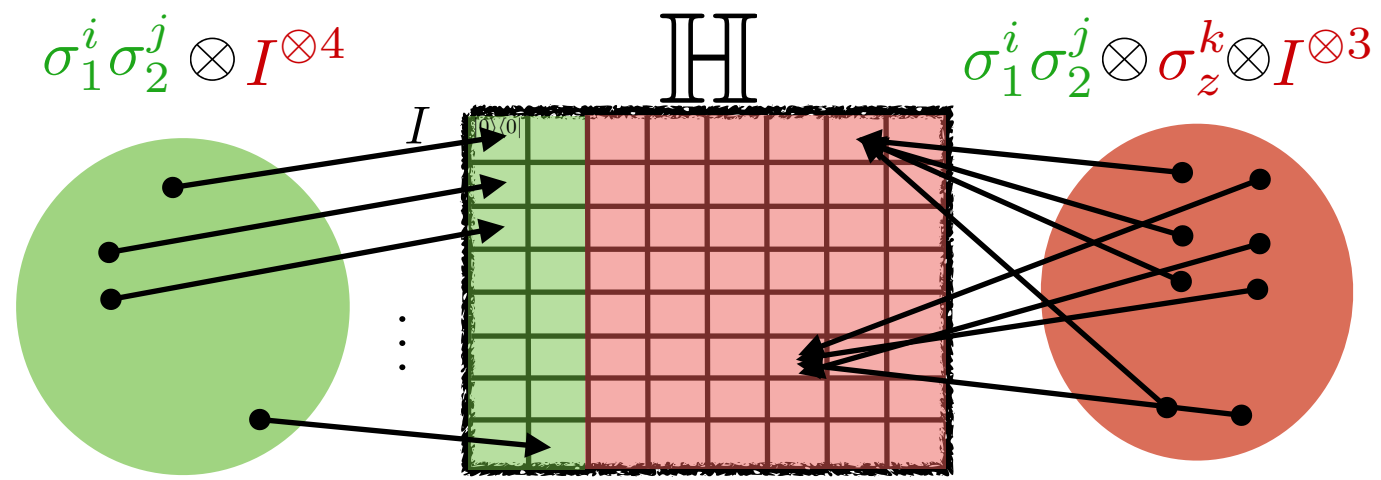


# Selectivity Reduced Noise



$i$	$E_i$	$e_i$	$i$	$E_i$	$e_i$
0	11	0000	8	Z1	1000
1	1X	0001	9	ZX	1001
2	1Z	0010	10	ZZ	1010
3	1Y	0011	11	ZY	1011
4	X1	0100	12	Y1	1100
5	XX	0101	13	YX	1101
6	XZ	0110	14	YZ	1110
7	XY	0111	15	YY	1111

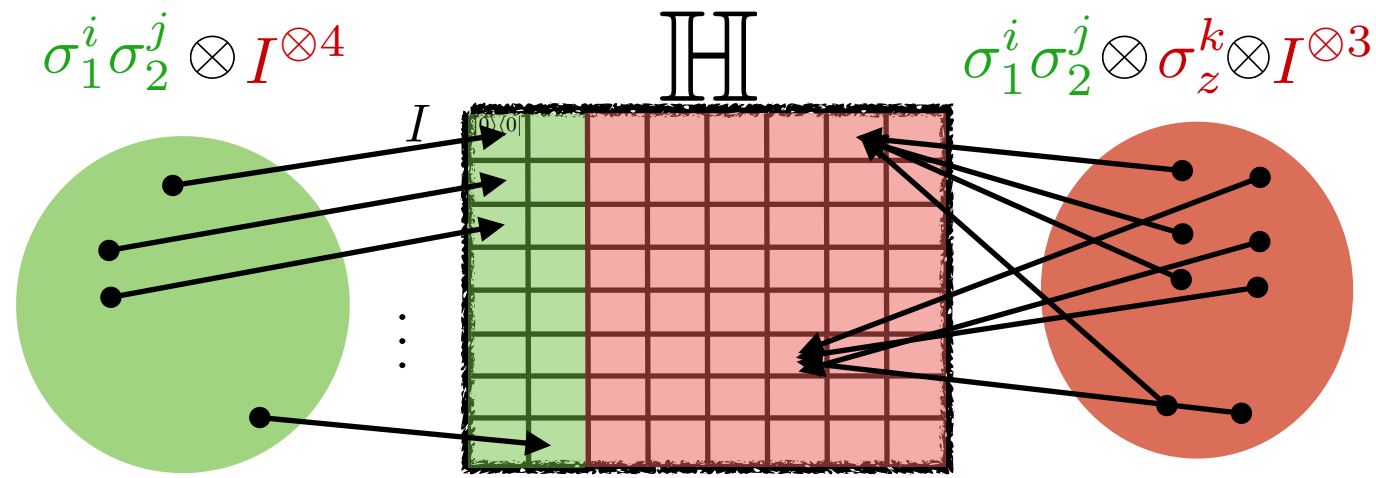
# Selectivity Reduced Noise



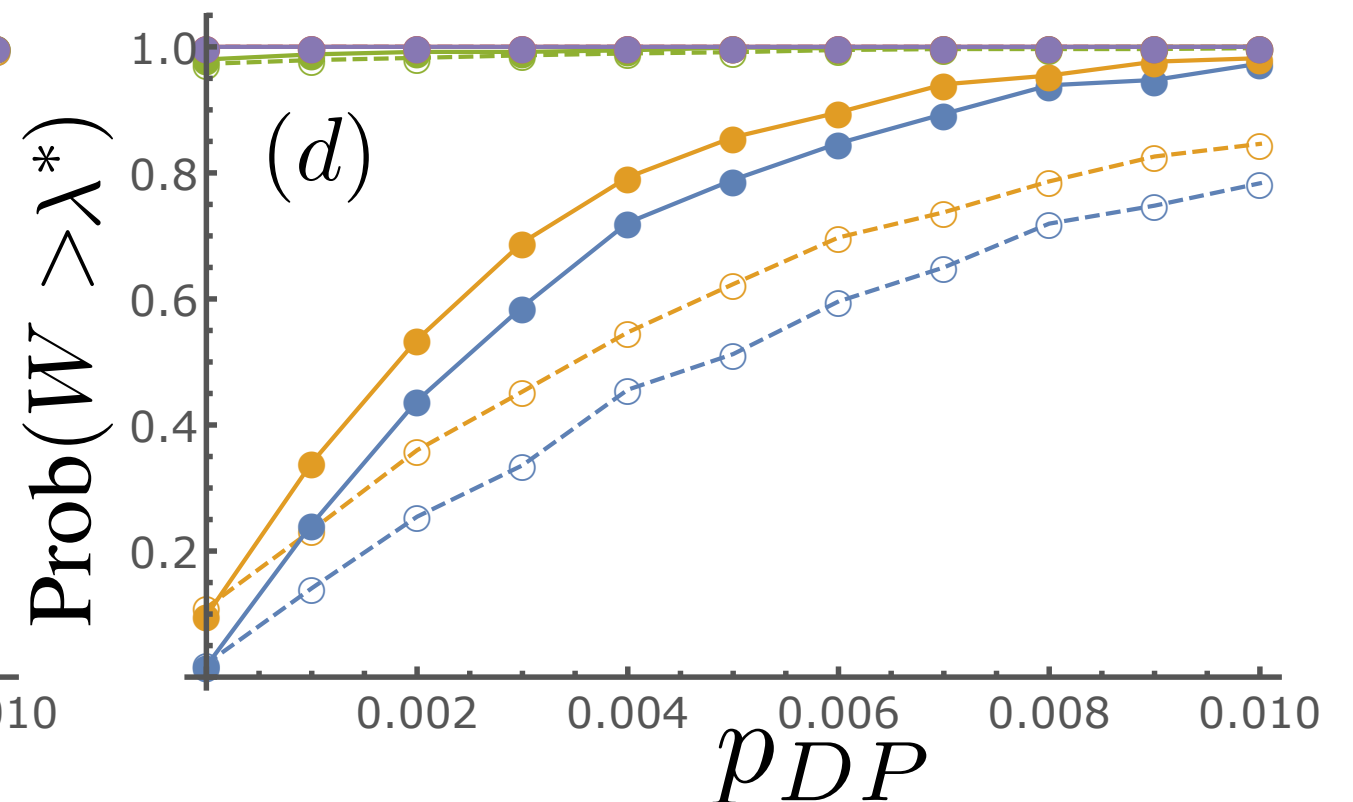
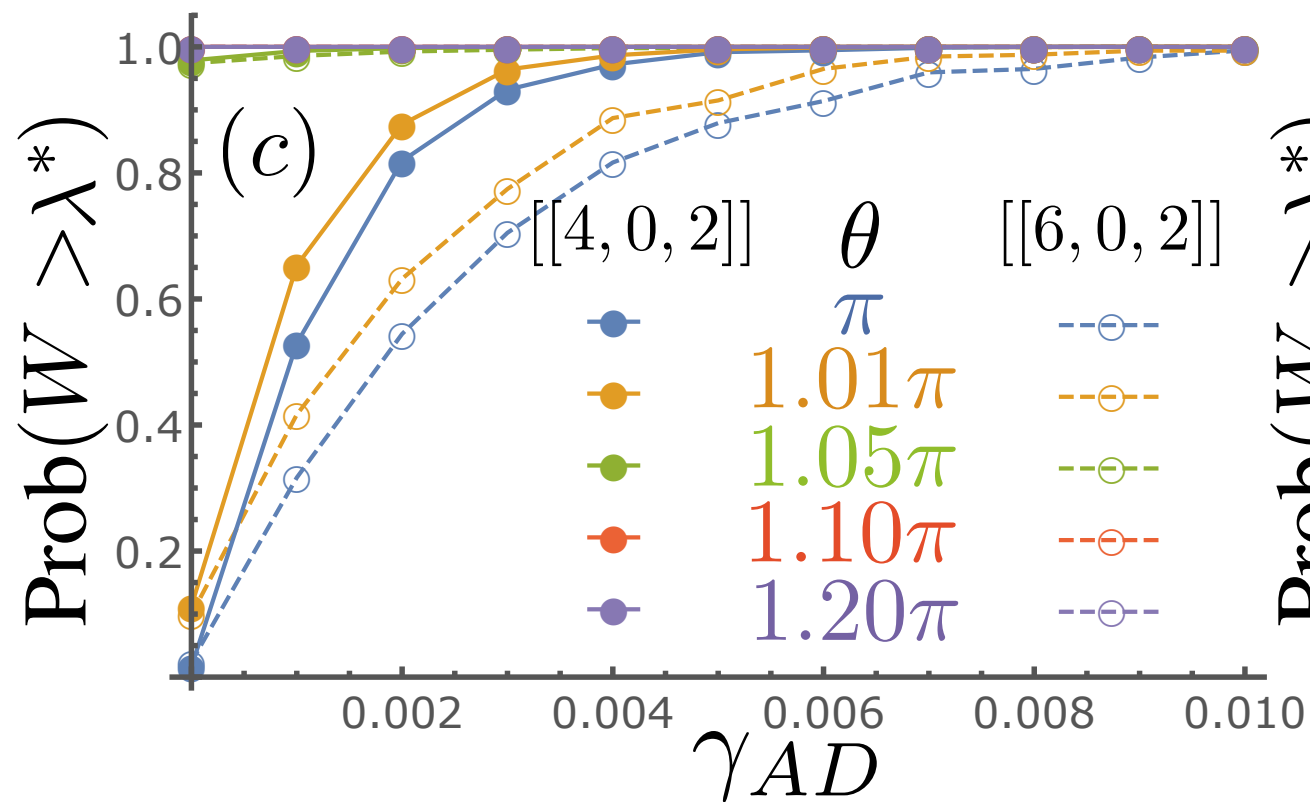
$i$	$E_i$	$e_i$	$i$	$E_i$	$e_i$
0	11	(00)0000	8	Z1	(00)1000
1	1X	(00)0001	9	ZX	(00)1001
2	1Z	(00)0010	10	ZZ	(00)1010
3	1Y	(00)0011	11	ZY	(00)1011
4	X1	(00)0100	12	Y1	(00)1100
5	XX	(00)0101	13	YX	(00)1101
6	XZ	(00)0110	14	YZ	(00)1110
7	XY	(00)0111	15	YY	(00)1111



# Selectivity Reduced Noise



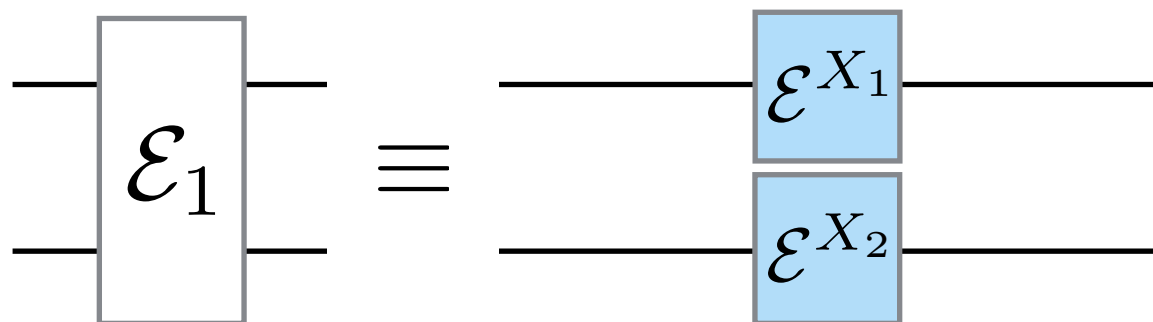
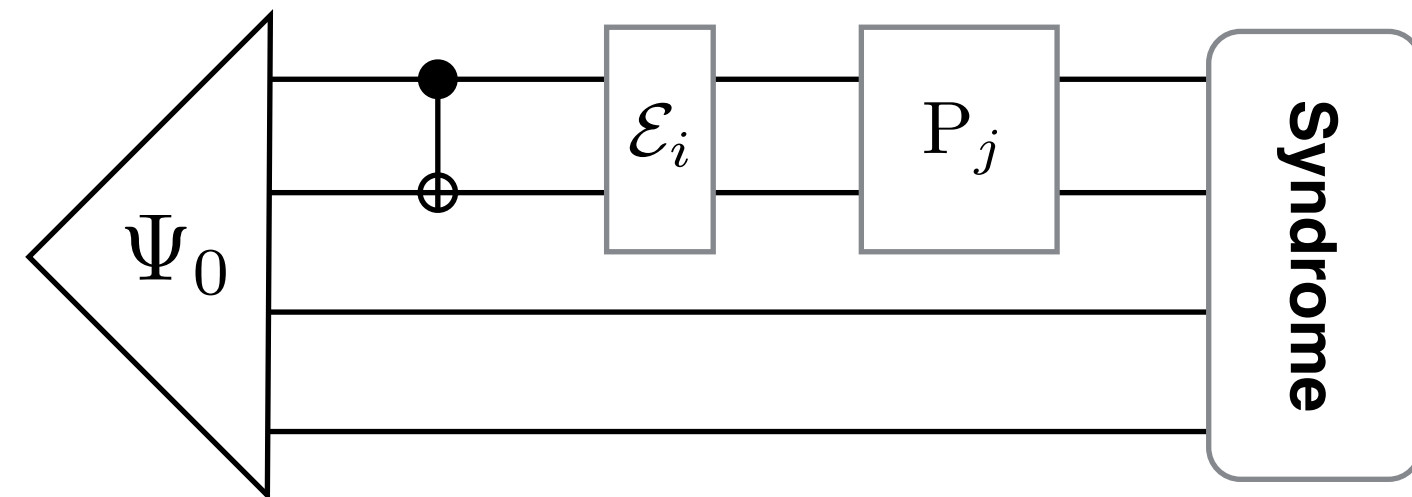
$i$	$E_i$	$e_i$	$i$	$E_i$	$e_i$
0	11	(00)0000	8	Z1	(00)1000
1	1X	(00)0001	9	ZX	(00)1001
2	1Z	(00)0010	10	ZZ	(00)1010
3	1Y	(00)0011	11	ZY	(00)1011
4	X1	(00)0100	12	Y1	(00)1100
5	XX	(00)0101	13	YX	(00)1101
6	XZ	(00)0110	14	YZ	(00)1110
7	XY	(00)0111	15	YY	(00)1111



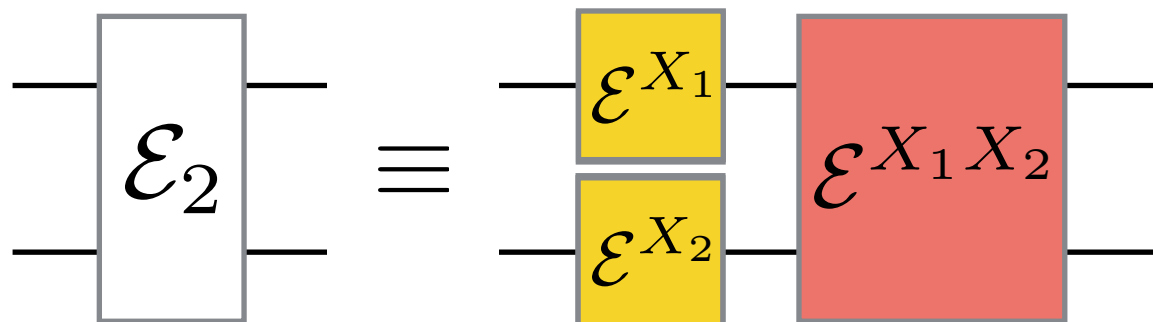
# Model #2: Bit-flip Crosstalk



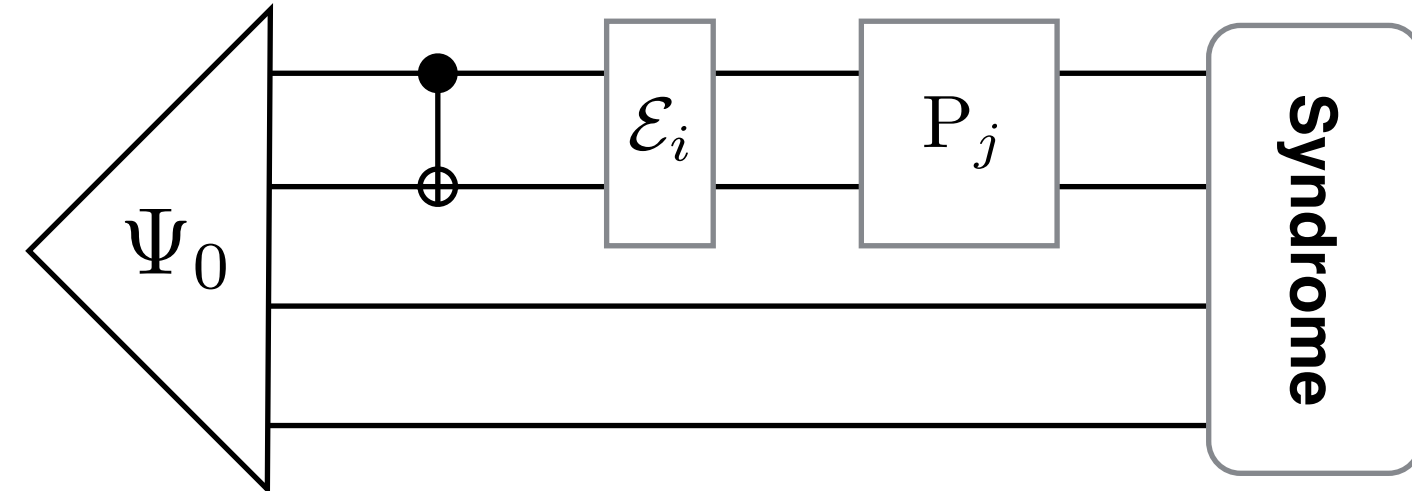
# Model #2: Bit-flip Crosstalk



$\mathcal{E}_{ind}$     $\mathcal{E}_{cor}$



# Model #2: Bit-flip Crosstalk



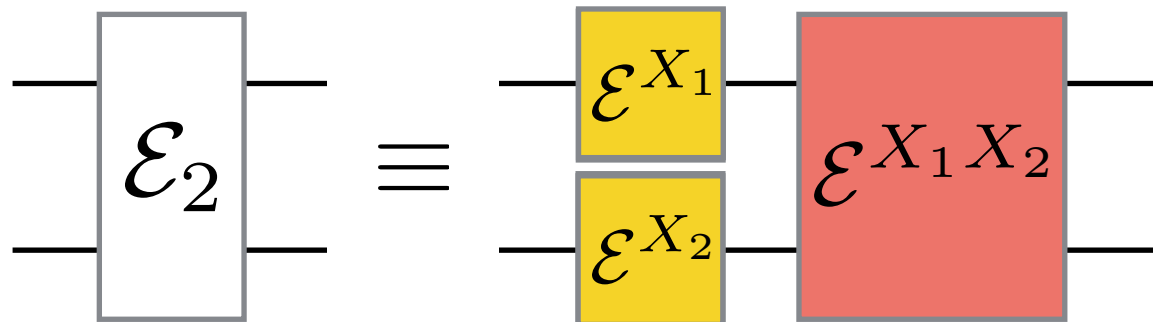
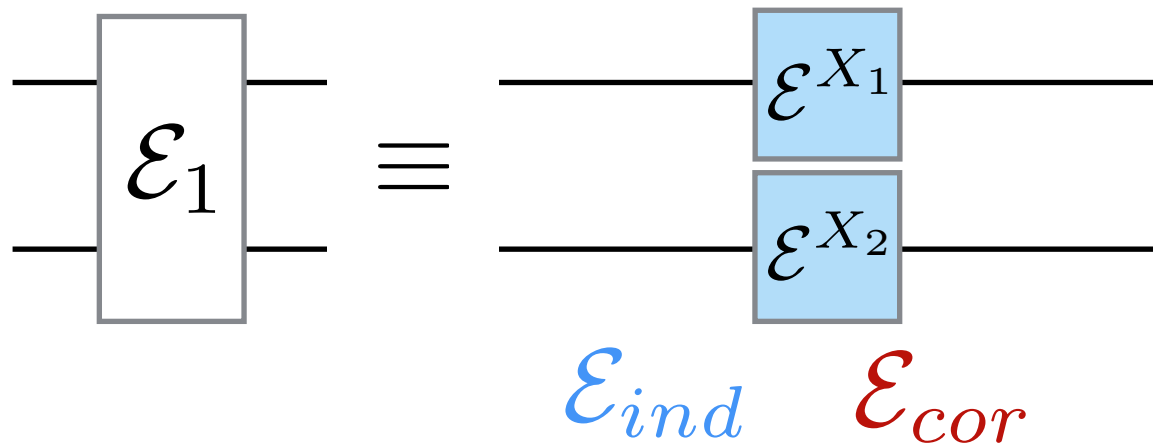
$$\mathcal{E}(\rho) = \sum_{m,n} \chi_{mn}^{\uparrow} F_m^{\uparrow} \rho F_n^{\uparrow\dagger} + \sum_{m',n'} \chi_{m'n'}^{\downarrow} F_{m'}^{\downarrow} \rho F_{n'}^{\downarrow\dagger}$$

$$\vec{F}^{\uparrow} = (II, ZI, IX, ZX)$$

$$\vec{F}^{\downarrow} = (XI, YI, XX, YX)$$

$$\chi^{\uparrow} = \frac{1}{4} \begin{pmatrix} \alpha + \beta & \beta - \alpha & \alpha + \beta & \alpha - \beta \\ \beta - \alpha & \alpha + \beta & \beta - \alpha & -\alpha - \beta \\ \alpha + \beta & \beta - \alpha & \alpha + \beta & \alpha - \beta \\ \alpha - \beta & -\alpha - \beta & \alpha - \beta & \alpha + \beta \end{pmatrix}$$

$$\chi^{\downarrow} = \frac{1}{4} \begin{pmatrix} \gamma + \delta & \gamma + \delta & \gamma - \delta & \delta - \gamma \\ \gamma + \delta & \gamma + \delta & \gamma - \delta & \delta - \gamma \\ \gamma - \delta & \gamma - \delta & \gamma + \delta & -(\gamma + \delta) \\ \delta - \gamma & \delta - \gamma & -(\gamma + \delta) & \gamma + \delta \end{pmatrix}$$



$$\alpha = p_1^X p_2^X p_{12}^X + (1 - p_1^X)(1 - p_2^X)(1 - p_{12}^X)$$

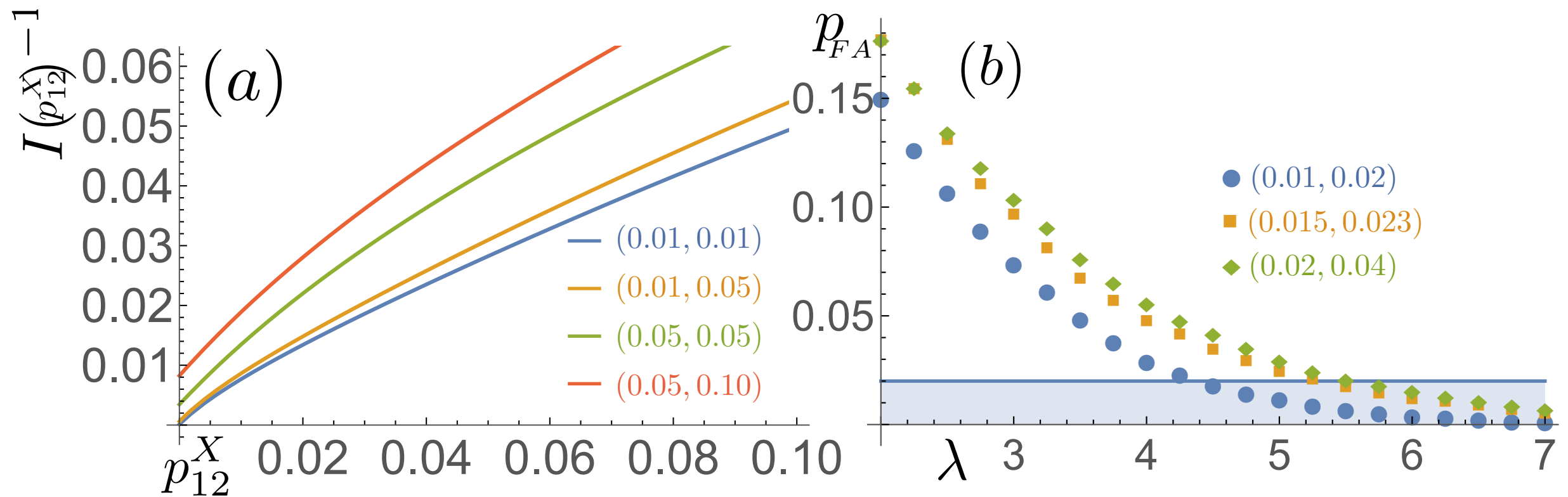
$$\beta = (1 - p_1^X)(1 - p_{12}^X)p_2^X + p_1^X(1 - p_2^X)p_{12}^X$$

$$\gamma = (1 - p_2^X)(1 - p_{12}^X)p_1^X + (1 - p_1^X)p_2^X p_{12}^X$$

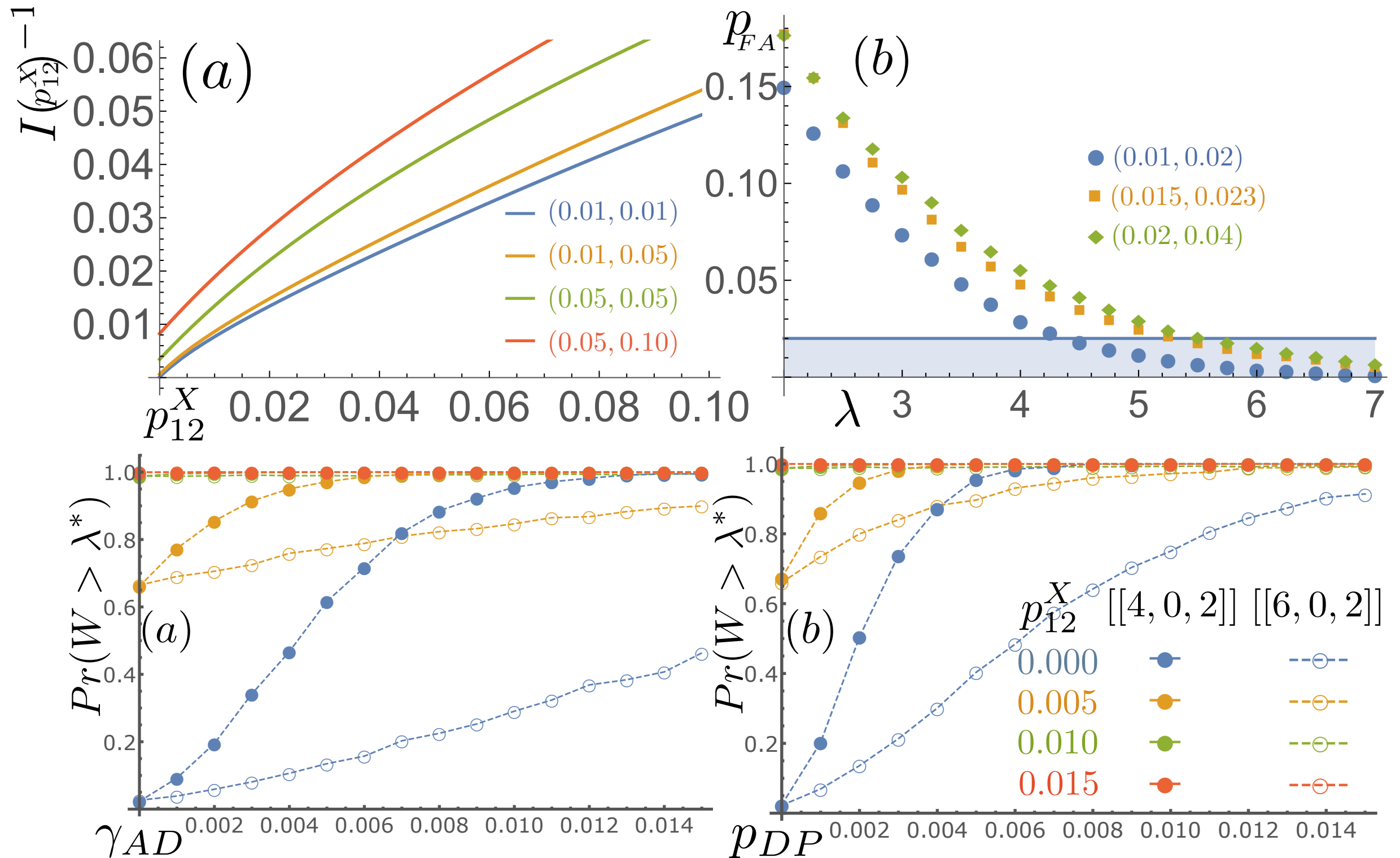
$$\delta = p_1^X(1 - p_{12}^X)p_2^X + (1 - p_1^X)(1 - p_2^X)p_{12}^X$$

# Bit Flip Inference

# Bit Flip Inference



# Bit Flip Inference



# Conclusions

# Conclusions

1. Developed direct process tomography quantum codes with error **detecting** codes with non-degenerate/degenerate bi-partite structure

# Conclusions

1. Developed direct process tomography quantum codes with error **detecting** codes with non-degenerate/degenerate bi-partite structure
2. Implemented statistical channel discrimination protocols by **selectively** probing relevant process elements

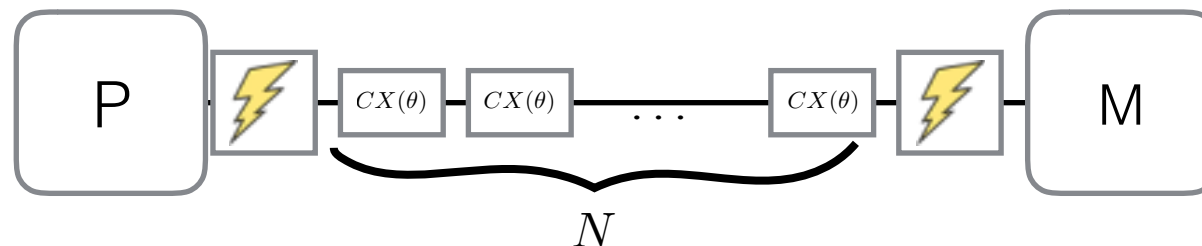


# Conclusions

1. Developed direct process tomography quantum codes with error **detecting** codes with non-degenerate/degenerate bi-partite structure
2. Implemented statistical channel discrimination protocols by **selectively** probing relevant process elements
3. We'd like to study if one can further develop sensing with quantum codes by also implementing adaptive (code) strategies? Relation to compressed sensing?

# Conclusions

1. Developed direct process tomography quantum codes with error **detecting** codes with non-degenerate/degenerate bi-partite structure
2. Implemented statistical channel discrimination protocols by **selectively** probing relevant process elements
3. We'd like to study if one can further develop sensing with quantum codes by also implementing adaptive (code) strategies? Relation to compressed sensing?
4. Develop SPAM error resistant protocol by studying discrimination characteristics when state subjected to a growing number of channel instances



Thank you!